I. INTRODUCTION TO MEASUREMENT PRACTICE

Objectives

The objective of this experiment is to introduce some basic concepts in measurement, and to develop good measurement habits. In the first section, we will develop guidelines for taking readings from instruments. In the second section, we will introduce the concept of uncertainty in measurement, and review significant figures and rounding rules. Finally, we will learn ways to compare readings and recognize discrepancies between measurements.

1. Guidelines for Taking an Instrument Reading

We begin with the mechanics of reading an instrument. You might think that taking an instrument reading is a trivial task. However, there is actually a great deal of thought that goes into taking measurements, and it is easy to make a mistake. A misread instrument is the cause of many costly—even deadly—mistakes in the field; so do not take this task for granted.

When reading an instrument, there are three parts that must be reported. The first is the reading itself, called the nominal value of the measurement. The second is the units of the measurement. The third part is to estimate the uncertainty of the reading, due to the resolution of the device. We will refer to this uncertainty as the reading error or resolution error. We will describe these parts in the sections that follow.

1.1 Nominal Reading

Consider the instrument displays of Figure 1.1, one with a digital display (the mass scale) and one with an analog display (the thermometer). The nominal value is the number read directly off the instrument’s display. The procedure for taking these reading is simple: Record exactly what you read from the digital display. For the display of Figure 1.1(a), record 0.30 g. Note that a value of 0.3 g or 0.300 g is incorrect, because either value misrepresents the resolution of the device.
The thermometer reading takes a little more thought: the reading appears to be halfway between the two gradations (marks) 76 and 77 °F. Which do we choose? Because the scale is analog, we can observe the position of the mercury between the readings, so we could guess the next decimal is, say, 76.4 °F. Many scientists, in fact, follow the procedure of guessing the next decimal. But interpolating between gradations, as this is called, can be difficult. The gradations can be so narrow that guessing the next decimal place, like 76.2 or 76.3, would be hard to justify. Instead, we will adopt this, more conservative rule to reading analog devices: Read analog displays to the nearest marked gradation, or if possible, halfway between. So in Figure 1.1(b), we would record the reading as 76.5 °F.

WARNING: With analog displays, extra care must be taken to ensure an accurate reading. Consider the thermometer displayed in Figure 1.2. Because the indicator (the mercury) is behind the scale (the lines on the glass), you must read the instrument with your eye normal to the instrument's display. Otherwise you risk misreading the instrument, as demonstrated in the figure. This error is called Parallax Error. Some instruments have built-in features to eliminate this effect, like placing a mirrored surface behind the scale so the operator can align his or her eye with the scale.
1.2 Units

Never forget to record the units exactly as listed on the device. **Look out for multipliers:** Some instruments give readings that are multiplied by a factor, say, 1000. Sometimes the multiplication factor can be chosen by the user with a dial or switch of some kind (typically referred to as *gain* achieved by an amplifier), so check for this feature.

1.3 Reading Error

For physical measurements, the true value of a property cannot be measured exactly. You can count the number of jelly beans in a jar exactly (assuming you have not miscounted), because that property has a *discrete value*; but the weight of those jelly beans has a *continuous value* – you can think of it as having an infinite number of decimal places. How close a measurement of a property is to its true value depends in part on the measuring instrument’s resolution.

Look again at the mass scale of Figure 1.1(a). Assuming there are no errors in the device or the measurement process, the true value of the mass is within

\[ 0.295 \leq \text{mass} \leq 0.304 . \]

This is implied because the device will round any value in this range to 0.30, the closest reading on the display. It is standard to record this measurement as

\[ \text{mass} = 0.30 \pm 0.005 \text{ g} . \]

In the above format, the value 0.30 is called the *nominal value*. The second term, ± 0.005, is called the *reading error*. It is equal to one-half the *resolution* (the smallest division between readings, sometimes called the *least count* for digital devices).

The same result applies to the thermometer of Figure 1.1(b). If we write the reading error as one-half the resolution (often called *minimum gradation* for analog devices), we can record the measurement as

\[ T = 76.5 \pm 0.5 \, ^\circ\text{F} . \]

The error above implies that the temperature reading is between 76 and 77 °F, which is in fact what was observed\(^1\). So to summarize, we can define the *reading error* for a device (analog or digital) as

\[ \text{Reading Error} = \pm \frac{1}{2} (\text{least count or minimum gradation}). \] (1.1)

**WARNING:** *For some digital displays, the last decimal reads only certain values.* For example, if the display of Figure 1.1(a) were to read only to the nearest odd decimal (i.e., 2.9, 3.1, 3.3, 3.5, etc.), the resolution would be 2 g, so the reading error would be plus or minus half that, or ± 0.1 g.

\(^1\) As we said before, the most common, scientific method is to guess the next decimal place (say, 76.4). We are choosing a more conservative method here.
1.4 Final Check: Think about the result

Always ask, “Does the value make sense? Was this the value I expected?” A reading that does not make physical sense, or is not expected, may indicate a problem with the experiment or with the instrument.

Another reason to anticipate the value of a measurement is that instruments often read incorrectly when the measured quantity is out of range. A pressure gauge may read 0.0 psi even though the actual pressure is lower, simply because the scale does not read below zero. A weight scale may “top out” at its full-scale reading, even if the weight on the scale is greater. This effect occurs in many digital instruments as well, so it is important to always be aware of the operational range of any instrument you read.

Example 1. Record the nominal value, resolution, reading error, and units for the following instruments:

a. 

\[\begin{array}{c}
\text{12.34 kg} \\
\text{Note: last digit reads only even values 0, 2, 4, 6, and 8.}
\end{array}\]

Solution: a. For a digital display, the nominal value is exactly as shown, which is 12.34. The resolution is the smallest division, which in this case is 0.02, since the last digit reads only even values. The uncertainty is always one-half the resolution, which in this case is \(\pm 0.01\). Finally, the unit is given as kg.

b. The needle in the analog display shown above appears to closest to halfway between 65 and 70, so we choose 67.5 as the nominal value. The resolution is the minimum division, which in this case is 5. The reading error is always one-half the resolution, which is \(\pm 2.5\) in this case. Finally, the unit is given as \(^{\circ}\text{C}\).

2. Measurement Uncertainty and Significant Figures

As we discovered in the previous section, all physical measurements have some level of uncertainty, defined as the inability to measure something exactly. The resolution of the measuring device is only one source of uncertainty; as we will discuss later in the course, the calibration of the device, the measurement process, and environmental conditions, to name a few, can also contribute to a measurement’s lack of accuracy.

The concept of significant figures is related to uncertainty. Consider the following measurement taken in a laboratory: 321.53 \(\pm\) 20 kg. Since the uncertainty of the measurement is \(\pm\) 20 kg, are the last three digits of the nominal value – 1.53 kg – necessary? After all, if the measurement were to the nearest 20 kg, then 1.53 kg would be impossible to resolve. In fact, only the first two digits of the nominal value are significant, and a more appropriate way to report this measurement would be as 320 \(\pm\) 20 kg.
Significant figures are important especially in engineering and scientific calculations, because often the uncertainties of measurements are not given. If a value of a measurement is given without an uncertainty, we will use the following rule to estimate the uncertainty:

**Rule for estimating uncertainty when none is given:**
If a number is given without an uncertainty, we will treat the number as if it was read off a digital display, and assume the uncertainty to be plus or minus one-half the least count.

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**Example 2.** Estimate the uncertainty of the following values: (a) 123.201, (b) 5334, and (c) $1.2345 \times 10^3$.

**Solution:** For estimation purposes, we assume the values above come from digital displays. Therefore,

a. the value 123.201 implies an uncertainty of ± 0.0005.

b. the value 5334 implies an uncertainty of ± 0.5.

c. the value $1.2345 \times 10^3$ implies an uncertainty of ± 0.00005 \times 10^3.

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### 2.1 Uncertainties in Calculations

A difficulty with measurements arises when performing calculations. For example, if we measured the length and diameter of a round tank, we could calculate the volume of the tank. If both the length and volume have an uncertainty, what is the uncertainty in the calculated volume? In this course, we will learn an analytical approach to estimating the resulting error, but as a first guess, we will rely on the technique we learned in Chemistry or Physics. This technique involves identifying the number of significant figures in the nominal values, and follows a set of rules for assigning the proper number of significant figures to the calculated result.

### 3 Review of Rules for Significant Figures

The rules for determining significant figures, and how they carry through calculations, are reviewed in the next section. We will then examine how significant figures are important in reporting measurements with uncertainties.

There are common rules for choosing the appropriate number of significant digits in a calculated result. These rules, which follow, are only a first step at evaluating the uncertainty of calculated values. A more rigorous and general method will be developed later in the course; until then, and as a first approximation, these rules will be used.

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Many texts argue that we should assume the last digit to be uncertain to plus or minus that digit; that is, 114 implies ± 1, while 20.23 implies ± 0.01. Approaches vary, and our approach, like all the others, is just an approximation.
Rules for Identifying Significant Figures

1. All non-zero numbers are significant.
   Examples: a. The value 237 has three significant figures.
              b. The value 123.45 has five significant figures.

2. Zeros between non-zero numbers are significant.
   Examples: a. The value 100.1 has four significant figures.
              b. The value 1027.0034 has eight significant figures.

3. Zeros used to locate the decimal point are not significant.
   Example: The value 0.000543 has three significant figures.

4. Zeros to the right of non-zero numbers are usually significant.
   Examples: a. The value 0.0230 has three significant figures.
              b. The value 1.200 \times 10^3 has four significant figures.
   Exception: The value 100 is ambiguous: is the measurement good to the nearest 1 unit, 10 units, or 100 units? This problem can be avoided using scientific notation, e.g., if the value were presented at 1.00 \times 10^2. Some texts suggest that including the decimal, i.e. “100.” implies that the zeros are significant. Or we could write the value as “100 to three significant figures.” In the meantime, it is probably best for calculation purposes if we assume three significant figures, but to make that assumption clear in your reporting.

5. Some values are considered exact, and therefore are not analyzed for significant figures.
   Examples: a. Conversion factors, such as 2.54 cm/in, are exact.
             b. There are 12 eggs in a dozen. This value is exact.

Example 3. Identify the number of significant figures in the following values: (a) 123.1, (b) 5.00, (c) 1.500 \times 10^3 (d) 1900

Solution: a. The value 123.1 has four significant figures, by Rule 1.
           b. The value 5.00 has three significant figures, by Rule 4.
           c. The value 1.500 \times 10^3 has four significant figures, by Rule 4.
           d. The value 1900 is ambiguous; it has either two, three, or four significant figures. To eliminate this ambiguity and imply four significant figures, this value could be written one of three ways:
              • 1.900 \times 10^3
              • 1900. (The period implies that all the zeros are significant.)
              • 1900 to four significant figures
Rules for Significant Figures in Calculations

1. Addition or subtraction: The resulting value should be reported to the same number of decimal places as that of the term with the least number of decimal places.

   Examples:
   a. 161.032 + 5.6 = 166.6 (166.632 rounded to one decimal place)
   b. 152.10 – 33.034 = 119.07 (119.066 rounded to two decimal places)

2. Multiplication or division: The resulting value should be reported to the same number of significant figures as that of the term with the least number of significant figures.

   Examples:
   a. 152.06 × 0.24 = 36 (36.4944 rounded to two significant figures)
   b. 1.057×10³ / 19.5 = 54.2 (54.205128… rounded to three significant figures)

3. When calculating averages or standard deviations, it is customary to add a significant figure to the result.

   Example: The average of 34.3, 20.0, 77.8, and 16.6 is reported as 37.18.

This seems like a lot of work, and it is at first, but it will become second nature. One common question is: What happens when there is more than one calculation in a problem? Do we have to follow these rules all the way through? The answer is no. In fact, the strict use of these rules through multiple calculations can cause severe errors due to rounding. Instead, most texts recommend that you carry an extra significant figure through your intermediate calculations. Many engineers carry more, or even all significant figures until the end of the calculation, then backtrack through the calculations to assign the proper number to the final result.

4. Rules for Reporting Nominal Values and Uncertainties with Proper Significant Figures

   In the previous section, we reviewed how to recognize significant figures, and described how they are carried through mathematical calculations. Notice, however, that the discussion thus far has been limited to measurements that had no explicit uncertainties – we inferred a level of accuracy by counting the significant digits. In experiments, you will be reporting measurements (and calculated results) with uncertainties. How many significant figures should you use? As you can probably guess, there are rules for reporting significant figures of nominal values and uncertainties. Again, the rules vary among texts and professions; we will adopt the following rules for this course:

   1. Limit uncertainties to one or two significant figures.

   We invoke this limit because any more than two significant figures in an uncertainty are usually negligible. For example, consider the reported measurement 123456 ± 6543 m. The first digit of the uncertainty represents 6000 m, or 4.9 percent of the nominal measurement. The next digit, or 500 m, is only 0.4% of the nominal value. The next digits are even smaller. This is illustrated in Figure 1.3. The point is that if the measurement is uncertain by about 6500 meters, what difference does an extra 43 meters make?
The nominal value of a measurement and its uncertainty should usually be written to the same decimal place.

This rule extends naturally from the previous rule: Consider again the measurement 123456 ± 6543 meters. Correcting its uncertainty, this measurement is expressed as 123456 ± 6500 meters. Since the uncertainty in the measurement is, at best, to the 100s place, then it makes no sense to report the nominal value to any greater accuracy. Thus the appropriate way to report this measurement is 123500 ± 6500 meters. (Note that this format is consistent with the addition/subtraction rule for significant figures of Section 3.2. That is, if we add or subtract the uncertainty to the nominal value, we would expect to reduce the digits of the result to match the least accurately-known value.)

NOTE: The decimals of the nominal value and the uncertainty do NOT always have to match exactly. Consider the digital reading shown in Figure 1.1a, whose nominal value is 0.30 g. We interpreted the uncertainty as ±0.005 g, so the reading should be expressed as 0.30 ± 0.005, NOT 0.030 ± 0.005. Adding an extra decimal place to the nominal value is misleading, because it gives the mistaken impression that the device was read to three decimal places.

For clarity, use the same notation (scientific or standard) for nominal values and uncertainties.

Examples:  

- a. $1.234 \times 10^5 \pm 300$ m should be written as $(1.234 \pm 0.003) \times 10^5$ m or $123400 \pm 300$ m.
- b. $5.68 \times 10^{-7} \pm 3 \times 10^{-9}$ kg should be written as $(5.68 \pm 0.03) \times 10^{-7}$ kg.
- c. $(213.01 \pm 0.23) \times 10^3$ m should be written as $(2.1301 \pm 0.0023) \times 10^5$ m. (Scientific notation always places the decimal after the first digit.)

Example 4. Correct the following reported measurements:

- a. $33.57 \pm 7.791$
- b. $45.667 \times 10^{-9} \pm 3.77 \times 10^{-10}$
- c. $10.20110 \pm 1.0988$
- d. $98.6 \pm 0.512$

Solution:  

- a. $33.6 \pm 7.8$. The uncertainty has two significant figures and its decimals match the nominal value’s.
- b. $(4.567 \pm 0.038) \times 10^{-8}$. The nominal value was first changed to proper scientific notation, $4.566 \times 10^{-8}$. Then the uncertainty was changed to match the scientific notation of the nominal value, $0.0377 \times 10^{-8}$. When these two values are combined, the uncertainty is reduced to two significant figures, and the number
of decimal places in the nominal value is changed to match the uncertainty as shown.
c. 10.2 ± 1.1.
d. 98.6 ± 0.5. The uncertainty was reduced to match the nominal value. (Little information is gained by keeping the second digit in the uncertainty.)

**Example 5.** A temperature measurement is reported as 25.3 ± 0.000034 °C. Does the measurement make sense? Explain.

**Solution:** At first glance, the reported uncertainty does follow the rules for significant figures. However, if the nominal value was read to only the tenths decimal place, as the value implies, then the reading error is approximately one-half the least count, or ± 0.05 °C. The reported uncertainty should be at least this amount, approximately. Instead, it is several orders of magnitude smaller. Either the nominal value is reported incorrectly or the uncertainty is misrepresented.

**Example 6.** Report the following instrument readings in proper format:

a. The instrument displayed in Example 1a.
b. The instrument displayed in Example 1b.
c. The digital mass measurement below.
d. The analog temperature measurement.
e. The analog temperature measurement.

**Solution:**

a. 12.34 ± 0.01 kg.
b. 67.5 ± 2.5 °C.
c. 0.530 ± 0.0025 kg.
d. 20.2 ± 0.05 °C

e. 20.45 ± 0.05 °C. Note that the nominal value is now taken to two decimal places. This is appropriate here because we could read the instrument to halfway between marks, or the nearest 0.05 °C. Be careful, though; if the uncertainty were left off this measurement, the extra decimal place would imply a better accuracy (± 0.005 °C) than intended.
5. Rules for Rounding Numbers

Finally, we will review the rules for rounding numbers. You probably were introduced to these in your Physics or Chemistry classes, but in the interest of completeness, they are included below.

<table>
<thead>
<tr>
<th>Rounding Rules</th>
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<tbody>
<tr>
<td>1. If the number to the right of the least significant digit is GREATER THAN 5, round the least significant digit UP.</td>
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<tr>
<td>Examples: 123456 rounded to four significant figures is 123500. 1.97832x10^5 rounded to three significant figures is 1.98x10^5. 4.99649 rounded to three significant figures is 5.00.</td>
</tr>
<tr>
<td>2. If the number to the right of the least significant digit is LESS THAN 5, round the least significant digit DOWN.</td>
</tr>
<tr>
<td>Examples: 123456 rounded to three significant figures is 123000. 1.97832x10^5 rounded to four significant figures is 1.978x10^5. 4.99649 rounded to four significant figures is 4.996. 3.49631 rounded to three significant figures is 3.50.</td>
</tr>
<tr>
<td>3. If the number to the right of the least significant digit is EXACTLY 5 (with no nonzero digits afterward), round the least significant digit UP if it is an ODD number; if it is EVEN, leave it unchanged.</td>
</tr>
<tr>
<td>Examples: 1235 rounded to three significant figures is 1240. 52500 rounded to two significant figures is 52000. 5.156500 rounded to four significant figures is 5.156.</td>
</tr>
<tr>
<td>Note: Why does this rule exist? Because if the least significant digit is exactly 5, we should, on average, round the number up half the time, and down half the time; otherwise we risk an accumulation of round-off error in repeated calculations (this is especially a problem with</td>
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**Example 7.** Round the value 1257.469575 to each successive decimal place, one at a time.

**Solution:** Beginning with the largest decimal place, the rounded values are as follows:

- 1000
- 1300
- 1260
- 1257
- 1257.5
- 1257.47
- 1257.470
- 1257.469
- 1257.4696 (Note: 7 is followed by exactly 5; since 7 is odd, it is rounded up)
- 1257.46958
- 1257.469575