1.2

a)

\[ s_1 = c(\text{rep}(0, 100), 10*\exp(-(1:100)/20)*\cos(2*\pi*1:100/4)) \]
\[ x_1 = \text{ts}(s_1 + \text{rnorm}(200, 0, 1)) \]
\[ \text{plot}(x_1) \]

\[ s_2 = c(\text{rep}(0, 100), 10*\exp(-(1:100)/200)*\cos(2*\pi*1:100/4)) \]
\[ x_2 = \text{ts}(s_2 + \text{rnorm}(200, 0, 1)) \]
\[ \text{plot}(x_2) \]
c) 

\[ \text{sma} = \exp\left(-\frac{1:100}{20}\right) \]

\[ \text{smb} = \exp\left(-\frac{1:100}{200}\right) \]

\text{ts.plot(sma)}

The signal in (a) looks similar to the earthquake signal, whereas the signal in (b) looks similar to the explosion signal.
1.3

a)

#plotting autoregression
w = rnorm(150,0,1)
x = filter(w, filter=c(0,-.9), method="recursive")[-(1:50)]
plot.ts(x, main="autoregression")

#adding moving average filter
v = filter(x, rep(1/4, 4), sides = 1)
#changing color of plot
ts.plot(x, v, col = c(1,2))
#adding dashed line to plot.
lines(v, lty = 2)
b)

```r
w = rnorm(100, 0, 1)
x = cos(2*pi*(1:100)/4)
v = filter(x, rep(1/4, 4), sides = 1)
ts.plot(cbind(x, v), col=c(1,2))
lines(v, lty = 2)
```

c)

```r
w = rnorm(100, 0, 1)
x = cos(2*pi*(1:100)/4) + rnorm(100,0,1)
v = filter(x, rep(1/4, 4), sides = 1)
ts.plot(cbind(x, v), col=c(1,2))
lines(v, lty = 2)
```
d)
The moving average smooths things out a lot. There is less volatility, and the trends stay closer to the mean.

1.4

\[
E \left[ (x_s - u_s)(x_t - u_t) \right] \\
E \left[ x_s x_t - x_s u_t - x_s x_t + u_s u_t \right] \\
E \left[ x_s x_t \right] = E \left[ x_s \right] u_t - u_s E \left[ x_t \right] + u_s u_t \\
E \left[ x_s x_t \right] = u_s u_t.
\]

\checkmark

1.5

s1 = c(rep(0,100), 10*exp(-(1:100)/20)*cos(2*pi*1:100/4))
x1 = ts(s1 + rnorm(200, 0, 1))
ts.plot(x1)
abline(h=mean(x1), col="purple")

compute locally
acf(x1)

Series x1

\[
s1 = c(rep(0,100), 10*exp(-(1:100)/200)*cos(2*pi*1:100/4)) = \text{ts(s1 + rnorm(200, 0, 1))}
\]
plot(x1)
abline(h=mean(x1), col="red")
\[ V(t) = E \left[ (w_{t-1} + 2w_t + (w_t + 1))^2 \right] \]
\[ = Ew_{t-1}^2 + 4w_t^2 + 2w_t + 1 = 6 \sigma_w^2. \]

\[ V(\theta, \theta + 1) = E (w_{t-1} + 2w_t + w_{t+1}) (w_{t+2} + 2w_{t+1} + w_{t+2}) \]
\[ = 4 \sigma_w^2. \]

\[ V(\theta, \theta + 2) = E \left[ (w_{t+1} + 2w_{t+2} + w_{t+3})^2 \right] \]
\[ = \sigma_w^2. \]

\[ V(\theta, \theta + h) = 0 \quad \text{for } h \geq 3. \]

\[ P(h) = \frac{V(t, t+h)}{V(t, t)} \]

\[ P(0) = 1 \]
\[ P(1) = \frac{2}{3} \]
\[ P(2) = \frac{1}{6} \]
\[ P(h) = 0 \quad \text{for } h \geq 3. \]
1.20

\[ w_{500} = \text{rnorm}(5000, 0, 1) \]
\[ \text{acf}(w_{500}, 20) \]

Series \( w_{500} \)

\[ w_{50} = \text{rnorm}(50, 0, 1) \]
\[ \text{acf}(w_{50}, 20) \]

Series \( w_{50} \)

If you add more obs, the confidence interval decreases. Less variance and volatility.
1.21

\[ w = \text{rnorm}(500,0,1) \]
\[ v = \text{filter}(w, \text{sides}=2, \text{filter}=\text{rep}(1/3,3)) \]
\[ \text{acf}(v[2:499], 20) \]

![Series v[2:499]](image)

It has more of a sin shape. Whereas the original ACF had a random shape.

1.23

\[ cs = 2\times\cos(2\times\pi\times 1:500/50 + .6\times\pi) \]
\[ w = \text{rnorm}(500,0,1) \]
\[ \text{acf}(cs+w, 100) \]

![Series cs + w](image)
acf(cs+w, 1000)

Series cs + w

The sin graph has a smaller velocity. The confidence interval remains the same.