ME 422 – Quiz 3
Winter 2016

In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.
Points: Problem 1 = 30% (a=15, b =15); problem 2 = 70% (a-e=12, g=10)

1. Draw the approximate root loci for the two systems shown below. On all paths draw arrow heads to show clearly how the closed-loop poles move as $K_p$ increases.

\[
\text{Mag}(F(s)) = \frac{K \cdot M_{z_1} \cdot M_{z_2} \cdots \cdot M_{z_m}}{M_{p_1} \cdot M_{p_2} \cdots M_{p_n}} = \frac{K \cdot \prod_{i=1}^{m} M_{z_i}}{\prod_{i=1}^{n} M_{p_i}} > 1
\]

\[
\text{Ang}(F(s)) = \theta_{z_1} + \theta_{z_2} + \cdots + \theta_{z_m} - \theta_{p_1} - \theta_{p_2} - \cdots - \theta_{p_n} = \sum_{i=1}^{m} \theta_{z_i} - \sum_{i=1}^{n} \theta_{p_i} = \pm 180^\circ
\]
2. The figure at right is the start of a root locus for a third-order system. The open-loop poles are shown as well as some of the closed-loop poles (square boxes) for a certain value of $K_p$. Answer the following questions in terms of the variables $a$, $b$, $c$, $d$ and $K_p$.

a. Finish the root locus for the system. In doing so, add any open- or closed-loop poles that are missing from the diagram. Show all calculations. If there are any special points, dimensions, asymptotes, angles, etc. to add, show them clearly.

b. Use the figure at right to answer this question. Use the closed-loop pole on the positive imaginary axis and draw the vectors you need to apply the angle and magnitude criteria there. Label these vectors ($M_{p1}$, for example) and their angles ($\theta_{p1}$, for example) so that you can clearly apply the magnitude criterion.

c. Write out the angle criterion in terms of $a$, $b$, $c$, and $d$.

\[ + \theta_{p1} + \theta_{p2} + \theta_{p3} = +180 \]

\[ \theta_{p1} = 90^\circ, \quad \theta_{p2} = \arctan \left( \frac{b+c}{a} \right), \quad \theta_{p3} = \arctan \left( \frac{c-b}{a} \right) \]

\[ \arctan \left( \frac{b+c}{a} \right) + \arctan \left( \frac{c-b}{a} \right) = 90^\circ \]
d. If $K_{st-a}$ without $K_P$ is 1, write out the magnitude criterion in terms of $a$, $b$, $c$, and $d$. Solve for $K_P$.

\[ \frac{K_P}{M_{p1} \cdot M_{p2} \cdot M_{p3}} = 1 = \frac{K_P}{c \cdot \sqrt{a^2 + (b+c)^2 \cdot \sqrt{a^2 + (c-b)^2}}} \]

\[ K_P = c \cdot \sqrt{a^2 + (b+c)^2 \cdot \sqrt{a^2 + (c-b)^2}} \]

e. On the time plot below, draw the unit step response ($y(t)$) of the closed-loop system as shown with the $K_P$ given for the pole locations in the first figure. Assume the closed-loop gain, including $K_P$, is $K_{st-a}$. Be as specific as you can in giving dimensions on the response graph, using $K_{st-a}, a, b, c, d$ as necessary. Show any calculations.

\[ w_c = \frac{2\pi}{L} \]

\[ P_d = \frac{2\pi}{w_c} = \frac{2\pi}{L} \]

f. What is a form of the characteristic equation in terms of $a$, $b$, $c$, and $d$? Multiply this out so that the equation is in a nice, clean, polynomial form.

\[ (s+d)(s+jc)(s-jc) = 0 \]

\[ s^3 + ds^2 + cs^2 + ds + c^2 = 0 \]