ME 422 – Quiz 1
Winter 2016

In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.

Points: c, f, k = 4; b, g, j, n = 5; e, h, i, l = 7; o=8; p=9; a, d, m=10.

1. A position control loop consists of a controller ($K_P$) and a second-order plant ($K_2$, $\omega_n$, $\zeta$). It has a first-order actuator ($K_1$, $T$). The second-order plant puts out a velocity that is then integrated up to a displacement. The control-loop is a unity-feedback loop.

   a. Draw this loop.

   b. What is the loop’s open-loop transfer function?

   c. What is the order of this $G_{OL}$? ___________________

   d. Figure out the ODE that corresponds to the transfer function $G_{OL} = \frac{x}{x_r}$. 
e. What is the closed-loop transfer function for this system?

f. What is the order of the closed-loop system? ___________________

g. What is the closed-loop steady-state gain?

h. Redraw the loop with velocity control instead of position control, using the same controller, actuator, and plant.

i. What is the closed-loop system’s transfer function \( (v/v_r) \)?

j. What is the closed-loop steady-state gain for this velocity control loop?

k. What is this closed-loop’s order? ___________________
l. Now remove the P-only controller and put in a PD controller with $K_P = 1$ and $K_D = T$. Draw this controller and get its transfer function.

m. What is the closed-loop transfer function of this PD-loop?

n. What is the PD-loop’s closed-loop steady-state gain?

o. What is its closed-loop natural frequency?

p. What is its closed-loop damping ratio?

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\begin{align*}
G_{cl} &= \frac{G}{1+GH} = \frac{N_G D_H}{D_G D_H + N_G N_H} \\
G_1 &= \frac{K_{ss}}{T s+1} \\
G_2 &= \frac{K_{ss} \omega_n^2}{s^2 + 2 \zeta \omega_n s + \omega_n^2} = \frac{K_{ss}}{\frac{s^2 + 2 \zeta \omega_n s + 1}{\omega_n^2}}
\end{align*}
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