Name ______________________________ Section 1510 1610

ME 422 – Quiz 1

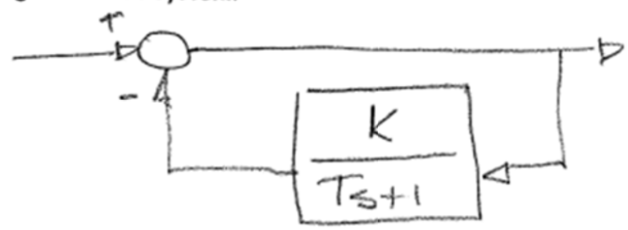
Winter 2011

In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.

1. A closed-loop system is formed as a unity-feedforward/non-unity-feedback system. In its feedback path is a conventional first-order block (K, T).

   a. Draw a block diagram of this system.

3



   b. What is the closed-loop transfer function?

2

$$G_{CL} = \frac{D_H}{D_H + N_H} = \frac{Ts+1}{Ts+1+K}$$

   c. What is the time constant of the closed-loop system?

2

$$G_{CL} = \frac{(Ts+1)/(1+K)}{\frac{T}{1+K}s + 1} \; ; \; \text{so } T_{CL} = \frac{T}{1+K}$$

   d. Calculate $c(\infty)$, the steady-state output of the closed-loop system, when it is subjected to a unit step input.

3

$$c(\infty) = \lim_{s\to 0} s \cdot G_{CL} \cdot R = \lim_{s \to 0} \cancel{s} \cdot \frac{(Ts+1)}{Ts+1+K} \cdot \frac{1}{\cancel{s}}$$

$$c(\infty) = \frac{1}{1+K}$$

2. A system has the ODE

$$5 \cdot \ddot{x} + 8 \cdot \dot{x} + 20 \cdot x = 0$$

The initial conditions $x(0) = 0$ and $\dot{x}(0) = 2$ are imposed.

a. What is the Laplace transform of the solution of this equation?

4

$$5\left(s^2 X(s) - s \cdot x(0) - \dot{x}(0)\right) + 8\left(s \cdot X(s) - x(0)\right)$$

$$\left(5s^2 + 8s + 20\right) X(s) = 5 \cdot 2$$

a. $$\boxed{X(s) = \frac{10}{5s^2 + 8s + 20} = \frac{2}{s^2 + 1.6s + 4}}$$

b. Will the response of this system be oscillatory or non-oscillatory? Explain your rationale clearly.

$$s^2 + 1.6s + 4 = s^2 + 2\zeta\omega_n s + \omega_n^2$$

5

$$\omega_n = 2, \quad 2\zeta\omega_n = 1.6 \Rightarrow \zeta = 0.4$$

Since $\zeta < 1$, oscillatory.

Answer *either* c1 or c2 but not both. Only one can apply, not both.

c1. Break the response up into simpler components and solve for *x*(*t*).

N/A

c2. Determine the oscillation frequency of the system. Show work.

2

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 2\,\frac{\text{rad}}{\text{sec}} \sqrt{1 - 0.4^2}$$

$$\omega_d = 2\,\frac{\text{rad}}{\text{sec}} \sqrt{1 - 0.16}$$

$$\boxed{\omega_d = \sqrt{0.84} \cdot 2\,\frac{\text{rad}}{\text{sec}} = 1.83\,\frac{\text{rad}}{\text{sec}}}$$

d. Calculate x(∞) with the given initial conditions.

2

$$x(\infty) = \lim_{s \to 0} s \frac{2}{s^2 + 1.6s + 4}$$

$$\boxed{x(\infty) = 0}$$