Name OWEN

Section ALL

ME 422

Quiz 1

4 October 2010

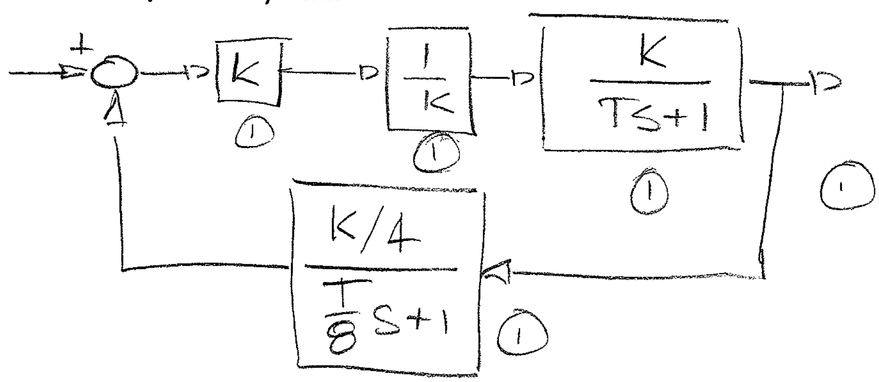
01 , 14.4, 3.6
02 , 13.5, 4.2

24pts total

Answer the problems below showing all work. A correct answer is insufficient for full credit. It must also be clear how you arrived at your answer.

1. A control loop has a P-only controller with a gain of K, an actuator with a gain of 1/K, and a first-order plant with K/(T·s + 1). The sensor is slow so is also modeled as a first-order system with (K/4)/[(T/8)·s + 1].

   a. Draw the control loop of this system.



(5)

   b. What is the open-loop transfer function of the control loop?

$$G_{OL} = \frac{K^2/4}{(Ts+1)\left(\frac{T}{8}s+1\right)}$$

$$= \frac{K^2/4}{\frac{T^2}{8}s^2 + \frac{9}{8}Ts + 1}$$ (2)

   c. What is the open-loop gain?

$$K_{SS-OL} = \frac{K^2}{4}$$ (2)

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d. What is the closed-loop transfer function of the control loop?

$$G_{CL} = \frac{N_G D_H}{D_G D_H + N_G N_H} = \frac{K\cancel{2}\left(\frac{T}{8}s+1\right)}{\cancel{K}(Ts+1)\left(\frac{T}{8}s+1\right) + K\cancel{2}\,K/4}$$

$$G_{CL} = \frac{K\left(\frac{T}{8}s+1\right)}{\frac{T^2}{8}s^2 + \frac{9}{8}Ts + 1 + K^2/4}$$ (4)

e. Give the closed-loop system's K_{ss}, ζ, and ω_n in terms of K and T. Show your work. (For a unit step input, K_{ss} is what is left after all the s's go to 0.)

$$G_{CL} = \frac{\frac{8K}{T^2}\left(\frac{T}{8}s+1\right)}{s^2 + \frac{9}{T}s + \frac{2(4+K^2)}{T^2}}$$ (2)

$$\omega_n = \sqrt{\frac{2(4+K^2)}{T^2}} = \frac{\sqrt{2(4+K^2)}}{T}$$ (2)

$$2\zeta\omega_n = \frac{9}{T}, \quad \zeta = \frac{9}{2T\omega_n} = \frac{9}{2}\frac{1}{\cancel{T}}\frac{\cancel{T}}{\sqrt{2(4+K^2)}}$$ (2)

$$\zeta = \frac{9}{2}\frac{1}{\sqrt{2(4+K^2)}}, \quad K_{ss\text{-}CL} = \frac{8K}{\cancel{T^2}}\frac{\cancel{T^2}}{2(4+K^2)}$$ (2)

$$K_{ss} = \frac{4K}{4+K^2}$$

f. Under what conditions would this system oscillate when given a step input? Make your reasoning and analysis clear.

$$\zeta = \frac{9}{2}\frac{1}{\sqrt{2(4+K^2)}} < 1$$

$$\frac{81}{4}\frac{1}{2(4+K^2)} < 1$$

$$\frac{81}{8} < 4 + K^2$$

$$\frac{81}{8} - 4 < K^2$$ (3)

$$K^2 > \frac{81-32}{8} = \frac{49}{8}$$

$$K > \frac{7}{2\sqrt{2}}$$