**Hurwitz stability criteria**

The Routh Hurwitz stability criteria involve the development of a so-called Routh array and then an inspection of it to determine whether there are right-half-plane poles and how many there are if they exist. One can use this method on systems of any order.

It is, however, rare to encounter systems of orders greater than 4 or 5 in industry. Higher order systems are usually simplified, with the fastest poles being neglected, since they play such a small role in the system response. Therefore a simplified set of rules can be applied for systems of order 5 or less. The rules follow.

For first and second order systems, the necessary and sufficient conditions for stability are that there is no change in signs of the coefficients of the characteristic equation. So the following systems are stable:

\[
\frac{16}{s^2+18s+52}, \quad \frac{-16}{s^2+18s+52}, \quad \frac{16}{-s^2-18s-52}, \quad \frac{16}{18s+52}, \quad \text{and} \quad \frac{16}{-18s-52}
\]

For third-order and higher systems, the situation is more complicated. For stable systems of all orders, there can be no change in signs of coefficients in the characteristic equation. This is a necessary condition. So no matter what the order, if there is a sign change in these coefficients, the system is unstable. But for systems of order three or higher, this is not a sufficient condition. That means there are, for example, third-order systems with no sign change in the coefficients of the characteristic equation, yet these systems are still unstable. For systems up to fifth order, we take the characteristic equation

\[a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0\]

**Sufficient Hurwitz conditions for stability:**

\(n = 3\): \(a_0 \cdot a_3 - a_1 \cdot a_2 < 0\)

\(n = 4\): \(a_0 \cdot a_2^2 + a_4 \cdot a_1^2 - a_1 \cdot a_2 \cdot a_3 < 0\)

\(n = 5\): \(a_2 \cdot a_5 - a_3 \cdot a_4 < 0\) and \((a_0 \cdot a_3 - a_1 \cdot a_2)^2 - (a_3 \cdot a_4 - a_3 \cdot a_5) \cdot (a_1 \cdot a_2 - a_0 \cdot a_3) < 0\)

So for systems of orders up to 5, for stability the systems need to meet both the necessary and sufficient conditions.

**Example**

See whether the following system is stable:

\[G_{CL}(s) = \frac{K}{s^3 + s^2 + s + 1}\]

Solution: All of the coefficients of the denominator have the same sign, so the necessary condition is met, and the system could be stable. Now check the sufficient condition. \(a_3, a_2, a_1, a_0 = 1\), so \(a_0 \cdot a_3 - a_1 \cdot a_2 = 0\), and this does not meet the sufficient condition for stability. So the system is
unstable. Alternatively, one could simply solve for the poles of the above system and find \( s = 13.4, -8.60 \pm j \cdot 1.707 \). Since there is a real root is in the right half-plane, the system is unstable. This real, unstable pole will lead to a positive power of an exponential term without a sine or cosine, so the system will explode monotonically. The stable pole pair will actually decay, swinging around this monotonic explosion. One can easily check this out with the Matlab step() command.

**Problem**

Put the above system into a unity feedback system with a proportional controller. Is the resulting system stable for certain values of \( K_p \)? If so, what are they?