ME 318 – Quiz 2
Summer 2013

In giving your answer, the answer alone is not enough. Make sure you clearly give your rationale for arriving at the answer. It must be clear to me how you arrive at your answer.
Problem weights: 1 = 70%, 2 = 30% each

1. An mck system has the following transfer function:

\[ T(s) = \frac{1}{0.05s^2 + 0.1s + 5} \]

a. What is \( \omega_n \)?

Need std form: \( \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \)

\[ \omega_n^2 = \frac{5}{0.05} = 100 \]

\[ \omega_n = 10 \text{ rad/sec} \]

b. What is \( \zeta \)?

\[ 2\zeta\omega_n = \frac{0.1}{0.05} = \frac{1}{5} = 0.2 \]

\[ \zeta = \frac{0.2}{2} = 0.1 \]

\[ \omega_n = 1, \quad \frac{\omega}{\omega_n} = 0.1 \]

\[ \zeta = \frac{1}{\omega_n} = 0.1 \]

c. Determine an expression for \( M(\omega) \).

Replace \( s \) with \( i\omega \)

\[ M(\omega) = |T(i\omega)| = \frac{1}{\sqrt{\left(5 - 0.05\omega^2\right)^2 + (0.1\omega)^2}} \]

\[ M(\omega) = \frac{1}{\sqrt{\left(5 - 0.05\omega^2\right)^2 + 0.01\omega^2}} \]

d. Determine an expression for \( \phi(\omega) \).

\[ \phi = 0 - \arctan \left( \frac{0.1\omega}{5 - 0.05\omega^2} \right) \]

\[ \phi = - \arctan \left( \frac{0.1\omega}{5 - 0.05\omega^2} \right) \]
e. The Bode plot for this system is shown below. Several points along the M curve are marked. Draw on the diagram the bandwidth for this transfer function. (Recall that \(20 \cdot log\left(\frac{1}{\sqrt{2}}\right) \approx -3 \, dB\). You can just draw on top of the boxes in the M diagram.

f. The system is excited by a forcing function \(f(t)\) that has the following Fourier approximation:

\[
f(t) \approx -0.2 \left[ \sin(3t) + \frac{1}{3} \sin(9t) + \frac{1}{5} \sin(15t) + \frac{1}{7} \sin(21t) \right]
\]

Draw the amplitude spectrum of this approximation of the forcing function on the plotting sheet below. Label axes and provide values for tick marks.
g. Calculate an approximate \( x(t) \), using only the components of \( f(t) \) that lie within the system bandwidth.

Only 1 component has a freq inside BW, namely the 2nd component @ 9 rad/sec.

\[
M(9) = 0.7642, \quad \frac{F_0}{2\pi} = 0.0667
\]

\[
x = M \cdot F_0 = 0.0509
\]

\[
\phi = -\arctan \left( \frac{0.109}{5-0.05 \cdot 9^2} \right) = -43.45^\circ
\]

which agrees with the Bode plot.

\[
\phi = 0.758 \text{ rad}
\]

So

\[
x(t) = -0.0509 \sin \left( 9t - 0.758 \right)
\]