ME 212 – Quiz 4
Winter 2013

Solve the problems below on this paper in the spaces provided. In your solutions you need to show not only the answers but the steps or rationale you used to arrive at the answer. If you perform special actions on your calculator (like a SOLVE or a cross product), write out the steps you used and precisely what you entered into the calculator. Your answers need to be complete enough to make your work checkable. Box your final answers. If you need more space, you may attach a paper with the continued part of the problem clearly designated as the continued part.

Points: \( a = 10 \), \( b = 6 \), \( c = 5 \), \( d = 2 \), \( e = 2 \), \( f = 4 \), \( g = 2 \), \( h = 6 \), \( i = 6 \), \( j = 11 \) = 54

1. To the right is a mechanism called a double pendulum. There is a real one in 13-101, the Mechanical Engineering Vibrations Lab. The pendulum consists of two bars, OA and AB. Both bars have mass \( m \) and length \( \ell \). Both bars are in motion as shown, swinging back and accelerating toward the middle position, where \( \theta_1 \) and \( \theta_2 = 0 \). In the configuration shown, obviously \( \theta_1 < \theta_2 \). At A, two unit vectors are shown, one perpendicular and one parallel to OA.

Answer the following questions.

a. Draw the FBD and MAD of the entire double pendulum on the drawing below. Use for the MAD accelerations parallel (n) and perpendicular (t) to each bar.

Need all 4 vectors because don’t know \( \ddot{a}_{AB} \) directly

\[ \ddot{a}_{AB} = \ddot{a}_{A-t} + \ddot{a}_{A-n} + \ddot{a}_{AB/A-t} + \ddot{a}_{AB/A-n} \]
b. Write the left-hand side of the equilibrium equation (the FBD part) below.

\[ \Sigma M_O = mg \frac{d}{2} \sin \theta_1 + mg \left( l \sin \theta_1 + \frac{d}{2} \sin \theta_2 \right) = \]

\[ \frac{1}{2} I \alpha_{OA} + I \alpha_{AB} \]

Do not list vectors with no moment about O. You may omit the moment arms.

c. List the vectors that would appear on the right-hand side of the equilibrium equation (the MAD part). You need only list them. You don’t have to write this half of the equation because the geometry is a little complicated.

\[ I \alpha_{OA}, I \alpha_{AB}, ma_{OA-t}, ma_{AB-t}, ma_{AB-n} \]

d. What is \( \hat{e}_1 \) in terms of \( \theta \) and \( i \) and \( j \)?

\[ \hat{e}_1 = -\sin \theta_1 \hat{x} + \cos \theta_1 \hat{y} \]

e. What is \( \hat{e}_2 \) in terms of \( \theta \) and \( i \) and \( j \)?

\[ \hat{e}_2 = -\cos \theta \hat{x} - \sin \theta \hat{j} \]

f. What is \( \bar{I} \)?

\[ \bar{I} = \frac{1}{12} ml^2 \]

g. Re-draw the MAD for the case where the position shown is actually the extreme position of the swing—i.e. where in the position shown, the pendulum has stopped and is starting to swing back toward the center.

MAD
h. Now go back to the original case where the pendulum is in motion. Draw the FBD and MAD of the top bar, using parallel and perpendicular components on the MAD.

Draw the FBD and MAD of the lower pendulum, making it consistent with the diagram in h. using also parallel and perpendicular components on the MAD. Don't forget to include the acceleration of point A.

1. Ask about acceleration of point A, in terms of 6.
   3 \( a_{A-n} = w_{OA}^2 \hat{e}_n = w_{OA}^2 \left( -\sin \theta_A + \cos \theta_A \right) \)
   3 \( a_{A-t} = \alpha_{OA} \hat{e}_t = \alpha_{OA} \left( -\cos \theta_A, \hat{t} - \sin \theta_A \right) \)