Five-term acceleration equation. \( P \) is a point moving on a rotating rigid body, and \( A \) is the origin of the rotating system \( xy \), which is attached to the body. The rigid body is moving in the inertial (fixed) reference frame \( XY \). The body’s rotation rate and angular acceleration are \( \omega_{AP} \) and \( \dot{\omega}_{AP} \).

\[
\ddot{a}_p = a_{AX} \hat{i} + a_{AY} \hat{j} \\
+ (a_{P/AX} - 2v_{P/AY} \omega_{AP} - r_{P/AY} \dot{\omega}_{AP} - r_{P/AX} \omega_{AP}^2) \hat{i} \\
+ (a_{P/AY} + 2v_{P/AX} \omega_{AP} + r_{P/AX} \dot{\omega}_{AP} - r_{P/AY} \omega_{AP}^2) \hat{j}
\]
At right is shown a pendulum consisting of a bar with mass $m$ and length $l$ and then a disk of the same mass $m$ but of radius $r$. The disk is rigidly attached to the bar halfway down its length, as shown. The assembly is rigid and moves together as a unit about the pivot point. Develop the equation of motion of the pendulum using rigid-body kinetics and Newton’s Second Law, by following the steps laid out below.

b. Now write the equilibrium equation that expresses the motion in terms of the variables shown above, $\theta$, and its derivatives.

\[
\begin{align*}
\sum M_0: & \quad -2mg \frac{r}{2} \sin \theta = 2ma + \frac{r}{8} + (I_D + I_R) \alpha \\
\alpha &= \dot{\theta}, \quad a = \frac{1}{2} \alpha, \quad I_D + I_R = \frac{1}{2} ml^2 + \frac{1}{12} ml^2 = m \left( \frac{r^2}{2} + \frac{l^2}{12} \right) \\
\theta'' + \frac{gl}{\left( \frac{l^2}{2} + \frac{r^2}{2} + \frac{l^2}{12} \right)} \sin \theta &= 0
\end{align*}
\]
At left is shown a spinning disk of radius $r$. It is spinning with the constant angular speed $\Omega$. A cockroach $C$ is walking across the spinning disk from point $A$ to point $B$ through a fixed point of rotation, point $O$, at a constant speed $v_x$ along a straight line on the disk. Answer the following questions about this situation.

a. Here is the five-term acceleration equation:

$$\ddot{a}_c = \ddot{a}_{ox} + \ddot{a}_{oy}i + (\ddot{a}_{c/0x} - 2v_c/0y\omega_{AB} - r_{c/0y}\omega_{AB}^2)i + (\ddot{a}_{c/0y} + 2v_c/0x\omega_{AB} + r_{c/0x}\omega_{AB}^2 - r_{c/0y}\omega_{AB}^2)j$$

Cross out the terms that disappear for this situation.

b. Plot the magnitude of the normal acceleration of the cockroach on the chart below as a function of position along the line of travel.

$$w_{MB} r$$

Indicate on the chart what the direction of normal acceleration is.

c. Plot the magnitude of the tangential acceleration of the cockroach on the chart below as a function of position along the line of travel.
Indicate on the chart what the direction of tangential acceleration is.  

\[ a_t \]

N/A

d. Plot the magnitude of the Coriolis acceleration of the cockroach on the chart below as a function of position along the line of travel.

Indicate on the chart what the direction of Coriolis acceleration is.

\[ a_{\text{cor}} \]

everywhere