Solve the problems below on this paper in the spaces provided. In your solutions you need to show not only the answers but the steps or rationale you used to arrive at the answer. If you perform special actions on your calculator (like a SOLVE or a cross product), write out the steps you used and precisely what you entered into the calculator. Your answers need to be complete enough to make your work checkable. Box your final answers. If you need more space, you may attach a paper with the continued part of the problem clearly designated as the continued part.

1. The slider-crank mechanism at right has the cylinder bore offset from the crank, so that the cylinder centerline does not intersect the crank axis at O. We want to analyze the motion of the mechanism in the configuration shown, when the connecting rod (AB) is vertical and the crank arm is at an angle \( \theta \). The crank arm is rotating counterclockwise at a constant angular velocity \( \omega_{OA} \).

Given are \( \omega_{OA}, l_{CA}, l_{AB}, \theta \). All of the answers you give to the questions below must be given in terms of these known parameters. If the answers you give contain other variables, they will not be considered correct.

a. Find \( \vec{v}_A \). Give it in terms of the unit vectors \( \hat{i}, \hat{j}, \) and \( \hat{k} \).

\[
\vec{v}_A = \vec{w}_{OA} \times \vec{r}_{A/O} = \omega_{OA} \hat{k} \times l_{OA} (\cos \theta \hat{i} + \sin \theta \hat{j})
\]

\[
\vec{v}_A = \omega_{OA} l_{OA} (-\sin \theta \hat{i} + \cos \theta \hat{j})
\]
b. Work out expressions for $\mathbf{v}_B$, $\mathbf{v}_{B/A}$, and $\mathbf{w}_{AB}$ in terms of the given quantities.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$$\mathbf{v}_{B/A} = \mathbf{w}_{AB} \times \mathbf{r}_{BA} = \mathbf{w}_{AB} \hat{k} \times \mathbf{r}_{AB} \hat{j} = -\mathbf{w}_{AB} \mathbf{r}_{AB} \hat{i}$$

$$\mathbf{v}_B \hat{j} = \mathbf{w}_{OA} \mathbf{r}_{OA} (-\sin \theta \hat{i} + \cos \theta \hat{j}) - \mathbf{w}_{AB} \mathbf{r}_{AB} \hat{i}$$

$$0 = -\mathbf{w}_{OA} \mathbf{r}_{OA} \sin \theta - \mathbf{w}_{AB} \mathbf{r}_{AB} \Rightarrow \mathbf{w}_{AB} = -\frac{\mathbf{w}_{OA} \mathbf{r}_{OA} \sin \theta}{\mathbf{r}_{AB}}$$

$$\mathbf{v}_B = \mathbf{w}_{OA} \mathbf{r}_{OA} \cos \theta$$

c. Draw a vector diagram of the relative velocity equation $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$. Show any important angles.

![Vector diagram](image)

d. Redraw the mechanism and identify its instant center.

![Redrawn mechanism](image)
e. Now perform an acceleration of the mechanism in the configuration shown. Find expressions for $a_B$ and $a_{AB}$, the magnitudes of $\vec{a}_B$ and $\vec{a}_{AB}$.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/\text{An}} + \vec{a}_{B/\text{At}}$$

$$\vec{a}_A = \omega_{OA} \times (\omega_{OA} \times \vec{r}_{A/C}) = \omega_{OA} \times \vec{v}_A$$

$$\vec{a}_A = \omega_{OA} \hat{k} \times \omega_{OA} \vec{l}_{OA} (-\sin \theta \hat{i} + \cos \theta \hat{j})$$

$$\vec{a}_A = -\omega_{OA}^2 \vec{l}_{OA} (\cos \theta \hat{i} + \sin \theta \hat{j})$$

$$\vec{a}_{B/\text{An}} = -\omega_{AB}^2 \vec{l}_{AB} \hat{j}$$

$$\vec{a}_{B/\text{At}} = \vec{a}_{AB} \times \vec{r}_{B/\text{At}} = \alpha_{AB} \hat{k} \times \vec{l}_{AB} \hat{j}$$

$$\vec{a}_{B/\text{At}} = -\alpha_{AB} \vec{l}_{AB} \hat{i}$$

Putting this all together:

$$\vec{a}_B \hat{j} = -\omega_{OA}^2 \vec{l}_{OA} (\cos \theta \hat{i} + \sin \theta \hat{j}) - \omega_{AB}^2 \vec{l}_{AB} \hat{j}$$

$$- \alpha_{AB} \vec{l}_{AB} \hat{i}$$

$$\alpha_{AB} = -\frac{\omega_{OA}^2 \vec{l}_{OA} \cos \theta}{\vec{l}_{AB}}$$

$$a_B = -\omega_{OA}^2 \vec{l}_{OA} \sin \theta - \omega_{AB}^2 \vec{l}_{AB}$$

$3$ of $3$