ME 212 – Quiz 2
Winter 2012

Solve the problems below on this paper in the spaces provided. In your solutions you need to show not only the answers but the steps or rationale you used to arrive at the answer. If you perform special actions on your calculator (like a SOLVE or a cross product), write out the steps you used and precisely what you entered into the calculator. Your answers need to be complete enough to make your work checkable. Box your final answers. If you need more space, you may attach a paper with the continued part of the problem clearly designated as the continued part.

1. 50% of quiz points

In the drawing at right a mass is located on an incline plane at rest at x=0, connected to an unstretched spring. The mass is let loose. Because the friction coefficient is low, the mass starts to slide down the incline plane until it comes to a stop before rebounding back upward.

a) Draw a free body diagram and mass acceleration diagram of the mass at some arbitrarily location x.

\[ F_{BD} = \begin{align*} m \ddot{x} &= F_k \\ N &= m \ddot{x} \\ \end{align*} \quad \text{MAP} \]

b) Impose equilibrium and develop an expression for the acceleration of the block in terms of the variables given in the drawing above.

\[ \begin{align*}
\text{1.} & \quad \sum F: \quad N - mg \cos \theta = 0, \quad N = mg \cos \theta \\
\text{2.} & \quad \sum F_x: \quad -F_k + mg \sin \theta + F_k = ma \\
\text{3.} & \quad -\mu mg \cos \theta + mg \sin \theta - \frac{Kx}{m} = ma
\end{align*} \]

\[ a = g (\sin \theta - \mu \cos \theta) - \frac{k}{m} x \]

1 of 3
c) Let $x = 0$ be the location where $V_x = 0$. Write the energy equation between the beginning state and the end state, where the beginning state is the block held at the start and let loose, and the end state is the block with the spring fully extended, at $x = x_{\text{max}}$.

\[ E_1 + U_{1-2} = E_2 \]

1. \[ F_1 + V_{g1} + V_{k1} + U_{1-2} = F_2 + V_{g2} + V_{k2} \]

2. \[-mg \cos \theta \cdot x_{\text{max}} = -mg x_{\text{max}} \sin \theta + \frac{1}{2} k x_{\text{max}}^2 \]

3. \[ mg (\sin \theta - \mu \cos \theta) = \frac{1}{2} k x_{\text{max}} \]

d) Find $x_{\text{max}}$ in terms of the variables in the drawing using the equation from c).

\[ x_{\text{max}} = \frac{2mg}{k} (\sin \theta - \mu \cos \theta) \]

7½ min
2. (30% of quiz points) Make sure you carefully show how you arrive at your answers. Answers without rationale are worth no points.

The masses shown have the velocities shown prior to impacting each other.

\[
\begin{align*}
m A v_A &+ m B v_B = m A v'_A + m B v'_B \\
m 2v + 2mv & = m v'_A + 2m v'_B
\end{align*}
\]

(a) Determine the final velocities \( v'_A \) (for \( m \)) and \( v'_B \) (for \( 2m \)) if \( e = 0 \).

\[
\begin{align*}
\epsilon = 0 &= \frac{v_B - v_A}{v_A - v_B} \\
\Rightarrow v'_B &= v'_A = v'
\end{align*}
\]

\[
\begin{align*}
\Box &= m v' + 2m v' = 3m v' \\
&= v'_A = v'_B
\end{align*}
\]

(b) Determine the final velocities — \( v'_A \) and \( v'_B \) — if \( e = 1 \).

\[
\begin{align*}
\epsilon = 1 &= \frac{v_B - v_A'}{v_A - v_B} = \frac{v_B' - v_A'}{2v - (-v)} \\
&= \frac{v_B' - v_A'}{3v}
\end{align*}
\]

velocity of approach

\[
\begin{align*}
v_B' - v_A' &= 3v \\
\Rightarrow v_B' &= 3v + v_A'
\end{align*}
\]

\[
\begin{align*}
2mv - 2mv &= m v_A' + 2m (3v + v_A')
\end{align*}
\]

\[
\begin{align*}
\Box &= v_A' (1+2) + 6v
\end{align*}
\]

\[
\begin{align*}
v_A' &= -2v \\
v_B' &= 3v + v_A' = v
\end{align*}
\]

7.24.30

7 1/2 min