Special notes on prob 12.28

This problem came up during office hours today, so I'd like to comment on it. It had to do with what to use to write equations for the rope length $\ell$ which was to be used for writing the equilibrium equations. My thinking's muddled here, so I'd like to show a solution method, maybe more than one. As always, you can pick any coordinate system you want, but you have to be consistent in applying it.

Solution method 1

With these defined this way, this means positive velocities & accelerations will be

\[ V_A, \alpha_A \quad V_C, \alpha_C \]

\[ V_B, \alpha_C \]
What you want to avoid is +x being one direction & +y or a being the other direction.

Rope length: \( l_{tot} = x_A + 2x_B + x_C + \text{lrett} \)

So \( 0 = v_A + 2v_B + v_C \) & \( 0 = a_A + 2a_B + a_C \)

Note that if \( v_B \) is positive (↑), \( v_A \) & \( v_C \) must be negative for this equation to be true.

With these directions established, directions are set for consistent axes as shown above. So when draw FBDs, use these coord systems.

**A:**

\[ A^+ \]

\[ \begin{array}{c}
\text{mg} \\
\text{DT} \\
\text{F}_{FA} \\
\text{N}_{A} \\
\end{array} \]

\[ m_{A} \]

\[ 2F_{x}: \quad -T + F_{FA} = m_{A}a_{A} \]

**B:**

\[ B^+ \]

\[ \text{mg} \]

\[ m_{B} \]

\[ a_{B} \]
\[ + \frac{1}{2} E_{Fx} : \quad m_b g - 2T = m_b a_b \]

\[ C : \quad T = F_{Fe} = m_c a_c \]

\[ - \frac{1}{2} E_{Fx} : \quad F_{Fc} - T = m_c a_c \]

**Matrix formulation:**

\[
\begin{bmatrix}
  m_a & 0 & 0 & 1 \\
  0 & m_b & 0 & 2 \\
  0 & 0 & m_c & 1 \\
  1 & 2 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
  a_a \\
  a_b \\
  a_c \\
  T
\end{bmatrix} =
\begin{bmatrix}
  F_{Fe} \\
  m_b g \\
  m_c a_c
\end{bmatrix}
\]

**Method 2:** Establish a central coordinate system and measure everything relative to it.

- \( X_0 \)
- \( Y_0 \)
- \( Z_0 \)

\[ X_0 A_0 X_0 \]

\[ Y_0 F_0 Y_0 \]

\[ Z_0 T_0 Z_0 \]

\[ G \]
Here \( x_A \) & \( y_B \) < 0 \& \( x_C > 0 \). So \(-x_A \) & \(-y_B\) are > 0.

\[
\begin{align*}
\mathbf{l}_{\text{tot}} &= -x_A - 2y_B + x_C + l_{\text{rest}} \\
\mathbf{O} &= -V_{A} - 2V_{B} + V_{C} \\
\mathbf{O} &= -a_{A} - 2a_{B} + a_{C}
\end{align*}
\]

If B moves down, \( V_{B} < 0 \) \& \(-2V_{B} > 0\).

With \(-V_{A} < 0 \) (A moves to right so \( V_{A} > 0 \)) \& \( V_{C} < 0 \) (C moves to left so \( V_{C} < 0 \)).

So velocity equation can be true. And acceleration follows same reasoning.

\( A_{0} \):

\[
\begin{align*}
\mathbf{8} \quad \mathbf{T} \\
\mathbf{F}_{\text{FA}}
\end{align*}
\]

\( \mp 2\mathbf{F}_{x} : \) \( T - \mathbf{F}_{\text{FA}} = m_{A} a_{A} \)

\( B_{0} \):

\[
\begin{align*}
\mathbf{2T} \\
\mathbf{m_{B} g}
\end{align*}
\]
\[ F_y = 2T - mg = ma \]

\[ T \rightarrow F_c \quad \rightarrow mca \]

\[ F_x = -T + F_c = mca \]

Organizing matrix:

\[
\begin{bmatrix}
  m_a & 0 & 0 & 0 & \Theta 1 \\
  0 & m_b & 0 & \Theta 2 \\
  \Theta 1 & 0 & m_c & 0 \\
  0 & \Theta 2 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
a_A \\ a_B \\ a_C \\ T
\end{bmatrix} =
\begin{bmatrix}
\Theta F_{FA} \\
\Theta mg \beta_g \\
\Theta F_{FC}
\end{bmatrix}
\]

Compare with previous result. Circled signs are different. Note that \( a_A \) here, let's call it \( a_{A2} \) is opposite in direction to \( a_{A1} \). And \( a_{B2} = -a_{B1} \). So if replaced these with \( -a_{B1} \) & \( -a_{B1} \), 1st two rows of method 2 are just 1st two rows of method 1 * -1. And it's legitimate to multiply an equation by -1.

3rd row is the same because assumed directions in both methods are the same.

In the last row, again if we replace \( a_{A2} \) with \( -a_{A1} \) & \( a_{B2} \) with \( -a_{B1} \), equation
is then the same.

I like method 1 more. It's less confusing to me. But you might like 2 better.

There are undoubtedly many other ways to do this. You may come up with a method you like better.

I believe the moral of the story here is to use a consistent coordinate system. That is the one you use for the net force equation you need also to use for the equilibrium equation.

You want \( \alpha \) in the kinematic equation (along...)

\[ \text{FBD} = \text{MAD equation}. \]