This homework set has seven (7) problems. Most of them are routine while some require more thought. You are encouraged to work with others and to ask questions of your instructor; however, you must write up your solutions independently. On this and all subsequent homework sets please write neatly and use complete sentences. Writing mathematics well is a craft, aim to hone it!

1. Show that the function \( f : \mathbb{R} \to \mathbb{R} \) given by
   \[
   f(x) = \begin{cases} 
   e^{-1/x^2} & \text{if } x \neq 0 \\
   0 & \text{if } x = 0 
   \end{cases}
   \]
is in \( C^\infty(\mathbb{R}) \), but is not real analytic. [Notice this example shows \( \text{R.A.} \subset C^\infty \).]

2. Show that \( \sin^2(x) + \cos^2(x) = 1 \) for every real number \( x \).

3. Show that \( e^x e^y = e^{x+y} \) for all real numbers \( x \) and \( y \).

4. Show that our function \( \sin \) is invertible on the interval \([−\pi/2, \pi/2]\). Give its inverse an appropriate name and show that the inverse is differentiable on its domain. Can you find a formula for \( (f^{-1})' \)?

5. Define \( \tan : (−\pi/2, \pi/2) \to \mathbb{R} \) by
   \[
   \tan(x) = \frac{\sin(x)}{\cos(x)}.
   \]
   Compute \( \tan'(x) \).

6. Show that our function \( \tan \) is invertible on the interval \((−\pi/2, \pi/2)\). Give its inverse an appropriate name and show that the inverse is differentiable on its domain. Can you find a formula for \( (f^{-1})' \)?

7. What is the largest neighborhood of \( c = 1 \) on which \( f(x) = \ln(x) \) is real analytic? Justify your assertion.