This homework set has seven (7) problems. Some of them are routine while others require more thought. You are encouraged to work with others and to ask questions of your instructor; however, you must write up your solutions independently. On this and all subsequent homework sets please write neatly and use complete sentences. Writing mathematics well is a craft, aim to hone it!

1. Find the pointwise limit for each of the sequences of functions defined below, then determine whether the convergence is uniform.
   
   (a) \( f_j(x) = jx(1-x)^j \quad x \in [0, 1] \)
   (b) \( f_j(x) = x^{2j}/(1 + x^{2j}) \quad x \in \mathbb{R} \)
   (c) \( f_j(x) = \sin jx/j \sqrt{x} \quad x \in (0, \infty) \)

2. Let \( \{f_j\} \) be continuous and converge uniformly on \([0, 1]\). Show that there is \( M \) such that \( |f_j(x)| \leq M \) for all \( j \) and all \( x \in [0, 1] \). Can you weaken the convergence to pointwise?

3. If \( f \) is continuous and \( f_j \to f \) uniformly on \( \mathbb{R} \), show that \( \lim_{j \to \infty} f_j(x + 1/j) = f(x) \forall x \).

4. Let \( f_j : [a, b] \to \mathbb{R} \) be continuous for all \( j \) and such that \( f_j \to f \) uniformly on \((a, b)\). Show that \( f \) may be extended to \([a, b]\) so that \( f_j \to f \) uniformly on the entire closed interval \([a, b]\).

5. \textit{RAF} Exercise 8.3.3

6. \textit{RAF} Exercise 8.2.5

7. \textit{RAF} Exercise 8.3.9