This homework set has six (6) problems. Some of them are routine, others require more thought, and a few you’ve done before. You are encouraged to work with others and to ask questions of your instructor; however, you must write up your solutions independently. Writing mathematics well is a craft, aim to hone your skill!

1. *Real Analysis & Foundations* 7.1.1

2. Prove Theorem 7.11(b) in *RAF*

3. *RAF* 7.1.2

4. *RAF* 7.1.4

5. Prove that the function \( f \) is Riemann integrable if and only if for all \( \varepsilon > 0 \) there exists \( \delta > 0 \) such that whenever \( \mathcal{P} = \{a, x_1, x_2, \ldots, x_{k-1}, b\} \) is a partition of \([a, b] \) with \( m(\mathcal{P}) < \delta \) we have

\[
\sum_{j=1}^{k} \left( \sup_{I_j} f - \inf_{I_j} f \right) \Delta_j < \varepsilon
\]

where \( I_j = [x_{j-1}, x_j] \).

[Your author provides some hints to both directions in *RAF* 7.1.5, 7.1.7. This result is a major step forward for us because it finally takes the “sample point ambiguity” out of Riemann integrability; if biggest and smallest Riemann sums can be made close together with fine enough mesh, then *any* Riemann sums can be made close together with fine enough mesh!]

6. Prove or disprove the following conjecture:

**Conjecture:** If \( f : [a, b] \to \mathbb{R} \) is bounded and \( \forall \varepsilon > 0 \) there exists a partition \( \mathcal{P} = \{x_0, x_1, x_2, \ldots, x_k\} \) such that

\[
\sum_{j=1}^{k} \left( \sup_{I_j} f - \inf_{I_j} f \right) \Delta_j < \varepsilon,
\]

then \( f \) is integrable on \([a, b] \).