Consider the following 15 questions an opportunity to demonstrate what you have learned so far. Each multiple choice question is worth 2 points and each free response question has points as marked. Read each problem carefully and write legibly. Show all of your work to receive full credit. Calculator use is prohibited. Please ask questions if you have any during the test. Good luck!

Problems 1-10 have exactly one correct answer each. Put the letter corresponding to your solution in the blank space on the right corresponding to each question.

1. The circumference of a circle of radius $r$ is:
   A. $\pi r^2$   B. $2\pi r$   C. $\pi r$   D. $2r$   E. none of the above
   
   1. B

2. The list $\{sit, hut, sum\}$ is:
   A. stable   B. tight   C. tight and stable   D. neither tight nor stable   E. none of the above
   
   2. B

3. The difference $d(\text{tool}, \text{loot})$ is equal to:
   A. 2   B. 3   C. 4   D. 1   E. 0
   
   3. A

4. The number of words on the shortest stable list that isn't tight is:
   A. 3   B. 2   C. 4   D. 1   E. 5
   
   4. C

5. The number $\pi$ is:
   A. irrational   B. rational   C. an integer   D. negative   E. less than 3
   
   5. A

6. You have a box of cookies, ten of which are chocolate chip and the rest are oatmeal. You randomly give ten cookies to your friend. The difference between the number of oatmeal your friend has and the number of chocolate chip you have left is:
   A. 10   B. 5   C. 3   D. 0   E. not enough information given
   
   6. D

7. The area of a triangle of base $b$ and height $h$ is:
   A. $bh$   B. $\frac{1}{2}bh$   C. $\frac{1}{2}bh^2$   D. $\frac{1}{2}b^2h$   E. none of the above
   
   7. B
8. A counterexample to the statement “If A, then B” is an example such that:
   A. B holds but A doesn’t  B. neither B nor A holds  C. both A and B hold  D. A holds but B doesn’t  E. not enough information
   8. \[ \text{D} \]

9. Fill in the blank: \[ \_ \equiv 7 \mod 18 \]
   A. 56  B. 24  C. 99  D. 79  E. none of the above
   9. \[ \text{D} \]

10. Fill in the blank: \[ 17 \cdot 26 \equiv \_ \mod 5 \]
    A. 0  B. 5  C. 3  D. 1  E. none of the above
    10. \[ \text{E} \]

Problems 11-13 each may have multiple answers. To get each question correct, you must list all correct answers.

11. The number \( \frac{5}{3} \) is a(n):
    A. positive whole number  B. integer  C. rational number  D. irrational number  E. real number
    11. \[ \text{C, E} \]

12. Between any two real numbers on the number line there lies a(n):
    A. natural number  B. integer  C. rational number  D. irrational number  E. multiple of 3
    12. \[ \text{C, D} \]

13. In *A Mathematician's Lament*, Lockhart compares teaching math to teaching:
    A. music  B. sports  C. art  D. spelling  E. yoga
    13. \[ \text{A, C} \]
14. (5 points) Prove or disprove: There exists a truce of four queens on the 4x4 chess board where one queen occupies a corner.

**FALSE**

**Pf:** Consider the 4x4 board

![4x4 chessboard with queen at a4](image)

If we put a queen Q in position a4, no queen can subsequently go in positions a1, a2, a3, b4, c4, d4, b3, c2 or d1. We are left to place three more queens but have only squares b1-b2-c1:

![Available squares](image)

or squares c3-d3-d2:

![Available squares](image)

available. Since this would require two queens on [image] or [image], which breaks the truce, a 4x4 truce is impossible. Note that putting a queen in any other corner follows by rotation. ✗
15. (5 points) Prove or disprove: If $a \equiv b \ (mod\ 4)$ and $a \equiv c \ (mod\ 4)$, then 4 divides $b - c$.

TRUE

Pf: Suppose $a \equiv b \ (mod\ 4)$ and $a \equiv c \ (mod\ 4)$.

This means $a-b = 4k$ for some integer $k$

and $a-c = 4j$ for some integer $j$.

Thus,

$b - c = (a-4k) - (a-4j) = 4(j-k)$.

That is, 4 divides $b - c$.  $\blacksquare$