Solutions to Review Problems on Derivatives

Note that the derivatives may not be fully simplified. 1. Find the derivative \( \frac{dy}{dx} (= f'(x)) \):

a) \( y' = f'(x) = \cos(\cos(\tan x)) \cdot (-\sin(\tan x)) \cdot \sec^2 x \)

b) \( y' = f'(x) = 12x \sec^6(x^2 + 1) \tan(x^2 + 1) \)

c) \( y' = f'(x) = \frac{(x^3+4x-9) \cos x-(3x^2+4) \sin x}{(x^3+4x-9)^2} \)

d) \( y' = f'(x) = -\frac{2(\tan(x^2-3)+2x^2 \sec^2(x^2-3))}{(x \tan(x^2-3))^3} \)

e) \( y' = f'(x) = (\sin 1)x^{(\sin 1-1)} \)

f) \( y' = f'(x) = 2x \)

g) \( y' = f'(x) = \frac{1-2xy^2}{2x^2 y+\cos y} \)

2. a) The slope of the line segment \( L \) is \( \frac{f(b)-f(a)}{b-a} \).

   b) Geometrically, this says that the tangent line at the point \( (c, f(c)) \) is parallel to \( L \).

   c) The right hand side represents your average velocity over the time interval \( a \leq t \leq b \). The left hand side represents your instantaneous velocity at time \( t = c \).

   d) The Mean Value Theorem guarantees that you went 100 km/hr at some point on your trip (maybe at many instants along the way, but definitely at least one instant).

3. a) \( f(x) \) is increasing where \( f'(x) > 0 \), i.e when \( x > 1 \). Similarly, \( f(x) \) is decreasing where \( f'(x) < 0 \), i.e. when \( x < 1 \).

   b) Based on part a) there is a local minimum at \( x = 1 \). There is no local maximum.

   c) \( f'(x) \) is increasing where \( f''(x) > 0 \) and \( f'(x) \) is decreasing where \( f''(x) < 0 \). Computing the second derivative, one obtains

   \[ f''(x) = (x-2)(3x-4) \]
Thus, \( f'(x) \) is decreasing on the interval \((4/3, 2)\) and \( f'(x) \) is increasing on \((-\infty, 4/3) \cup (2, \infty)\).

d) Concavity is determined precisely by the sign of the second derivative. Thus, by part c), \( f(x) \) is concave up on \((-\infty, 4/3) \cup (2, \infty)\) and concave down on \((4/3, 2)\).

e) An inflection point is where the concavity changes. By part d) this happens when \( x = 4/3 \) and \( x = 2 \).