Review Problems on Derivatives

1. Find the derivative $\frac{dy}{dx}(= f'(x))$:

   a) $y = f(x) = \sin(\cos(\tan x))$ b) $y = f(x) = (\sec^2(x^2+1))^3$

   c) $y = f(x) = \frac{\sin x}{x^3+4x-9}$ d) $y = f(x) = (x\tan(x^2-3))^{-2}$

   e) $y = f(x) = x^{\sin 1}$ f) $y = f(x) = \frac{x^4+2x^2+1}{1+x^2}$

   g) $x^2y^2 + \sin y = x$

2. The Mean Value Theorem (see text pp. 234-236) states:

   If f is a differentiable function on the interval $[a,\ b]$,then there exists a number c between a and b such that

$$f'(c) = \frac{f(b)-f(a)}{b-a} \quad \text{or, equivalently } f'(c)(b-a) = f(b)-f(a) \qquad (1)$$

The Mean Value Theorem is very important; it is a key element in our understanding of many other facts about calculus.

a) Draw a picture of a continuous function with domain some interval $[a,\ b]$. Draw the straight line segment L connecting the points $(a, f(a))$ and $(b, f(b))$ in your picture. What is the slope of the line segment L(in terms of a,b, and f)?

b) The derivative $f'(c)$ represents the slope of the tangent line to the graph at the point $(c, f(c))$. Find (at least) one point c in your picture where equation (1) is true. Interpret this equation geometrically.

c) Suppose we have a car moving along a straight highway. Its position (in km) at time x hours is given by the function $y = f(x)$. What does the right hand side of equation (1) represent physically? What does the left hand side represent physically?

d) Suppose you drive from here to Bakersfield along a straight line highway–more or less like Highway 99. Your average velocity is 100 km/hr. What does the Mean Value Theorem tell you happened at some time c during the trip?

3. Suppose we have function $y = f(x)$ for which $f'(x) = (x-1)(x-2)^2$.

   a) Where is the graph of $f(x)$ increasing? Decreasing?

   b) Where does $f(x)$ have a local maximum or minimum?

   c) Where is the graph of $f'(x)$ increasing? Decreasing?

   d) Where is the graph of $f(x)$ concave up? Concave down?

   e) Where does $f(x)$ have inflection points?