This homework set has seven (7) problems. Most of them are routine while some require more thought. You are encouraged to work with others and to ask questions of your instructor; however, you must write up your solutions independently. On this and all subsequent homework sets please write neatly and use complete sentences. Writing mathematics well is a craft, aim to hone it!

1. Formulate and prove a Theorem a la RAF3 Theorem 7.11 for Riemann-Stieltjes integrals.

2. Let $\alpha : [a, b] \to \mathbb{R}$ be increasing and $f : [a, b] \to \mathbb{R}$ be bounded. If $Q$ is a refinement of a partition $P$, prove that $L(f, Q, \alpha) \geq L(f, P, \alpha)$.

3. Evaluate $\int_{0}^{4} x^2 d\alpha$

   where $\alpha(x) = \lfloor x \rfloor$ is the greatest integer function.

4. RAF 7.3.2

5. Let

   $f(x) = \begin{cases} 
   0 & \text{if } 0 \leq x \leq 1 \\
   1 & \text{if } 1 < x \leq 2 
   \end{cases}$

   Show that $f$ is not Riemann-Stieltjes integrable with respect to itself on $[0, 2]$.

6. RAF 7.4.1

7. RAF 7.4.5