After our lengthy discussion on indexed families, it seems some follow up is in order. The following exercises are not ever due and need not be included in your portfolios. They are just for you to work on to solidify your feeling for these indexed families; remember, the indices are just notation to help us deal with families of sets in an orderly fashion.

1. Write out the indexed family $\mathcal{A} = \{A_\alpha : \alpha \in \{1, 2, 3, 4, 5\}\}$ “longhand”, that is, just list out the entire family.

2. Write out the indexed family $\mathcal{A} = \{A_\beta : \beta \in \{\spadesuit, \heartsuit, \diamondsuit\}\}$ “longhand” again.

3. Use indices to write the family $\mathcal{B} = \{B_1, B_2, B_3, B_4, \ldots, B_{100}\}$ in “short-hand”.

Exercise 3.44 in the text is to settle on what it means to take the union and intersection of an entire indexed family. If you already know what it means to take the union and intersection of an unindexed family, these should come as no surprise:

$$\bigcup_{\alpha \in \Delta} A_\alpha = \{x \in U : x \in A_\alpha \text{ for at least one } \alpha \in \Delta\}$$

and

$$\bigcap_{\alpha \in \Delta} A_\alpha = \{x \in U : x \in A_\alpha \text{ for every } \alpha \in \Delta\}.$$ 

4. Consider the indexed family $\mathcal{B} = \{B_v : v \in \mathbb{R}\}$ where $B_v = [v^2, v^2 + 1]$.

   (a) What is $B_5$?
   
   (b) What is $B_{\sqrt{2}}$?
   
   (c) Is there any $w \in \mathbb{R}$ so that $w \in B_w$?
   
   (d) Determine

   $$\bigcup_{v \in \mathbb{R}} B_v \text{ and } \bigcap_{v \in \mathbb{R}} B_v.$$ 

   (e) How would you prove your assertions from part (d)?
Let us now turn our attention away from calculations involving a specific family of sets like \( B \) above and toward more general statements concerning families of sets. For example, propositions 3.49-3.54 all fall into this latter category. Here, we have only the definitions to keep us afloat; if we stray, we sink.

5. Consider

**Proposition 3.51.** If \( A = \{ A_\alpha : \alpha \in \Delta \} \) is an indexed family of sets and \( B \) is a set, then

\[
\left( \bigcup_{\alpha \in \Delta} A_\alpha \right) - B = \bigcup_{\alpha \in \Delta} (A_\alpha - B).
\]

(a) What is the broad scale structure of the statement?
(b) What is the antecedent? The consequent?
(c) What, therefore, is the first line of your proof of Proposition 3.51? The last line?

With the structure well in hand, and only with the structure well in hand, we can now proceed to the details. Since the consequent is a set equality statement, we need to show subset containments both ways in order to conclude that

\[
\left( \bigcup_{\alpha \in \Delta} A_\alpha \right) - B = \bigcup_{\alpha \in \Delta} (A_\alpha - B).
\]

(d) What is the first line of the first half of the set equality argument for the consequent?
(e) What does

\[ x \in \left( \bigcup_{\alpha \in \Delta} A_\alpha \right) - B \]

truly mean?
(f) Now that there exists \( \alpha \in \Delta \) such that \( x \in A_\alpha \) and \( x \notin B \), can we conclude that

\[ x \in \bigcup_{\alpha \in \Delta} (A_\alpha - B)? \]

(g) Can you “reverse the argument” to now show

\[
\left( \bigcup_{\alpha \in \Delta} A_\alpha \right) - B \supseteq \bigcup_{\alpha \in \Delta} (A_\alpha - B)?
\]

(h) Write up your proof of Proposition 3.51.