Homework # 2, Math 143 Due Friday, October 11, 2013

This homework set has seven (7) problems. Some of them are routine, others require more thought. You are encouraged to work with others and to ask questions of your instructor; however, you must write up your solutions independently. On this and all subsequent homework sets please write neatly in complete sentences. Writing mathematics is a craft, aim to hone this skill!

1. Show that if \( a_n > 0 \) for every \( n \) and \( \sum a_n \) converges, then \( \sum \ln(1 + a_n) \) converges as well.

2. Add the first ten terms of the series \( \sum_{n=1}^{\infty} \frac{1}{n^3 + n^2} \). Give an upper bound on the error between this estimate \( S_{10} \) and the true sum.

3. Determine the true sum of the series
\[
\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}
\]
accurate to four decimal places.

4. Determine convergence or divergence of
\[
\sum_{n=1}^{\infty} (\sqrt{2} - 1)
\]

5. Sum the series
\[
\sum_{n=2}^{\infty} \ln\left(1 - \frac{1}{n^2}\right)
\]

6. Is the 10\(^{10}\)th partial sum \( S_{10^{10}} \) of the alternating harmonic series
\[
\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}
\]
an overestimate or an underestimate? Give an upper bound on the error in using this partial sum to approximate the true value.

7. Prove part (i) of the Root Test.