The linear objects in $\mathbb{R}^2$ are straight lines. One dimension higher, the linear objects in $\mathbb{R}^3$ are planes.

**Exercise 1** Draw the usual $xyz$-coordinate system with the $x$-axis protruding out of the page, the $y$-axis pointing to your right and the $z$-axis pointing “up”. Now shade in the $xy$-plane lightly, so that the coordinate axes are still visible (though some of the negative $z$-axis may be obscured now because it’s partly “below” your $xy$-plane.)

[Tricks of the trade include using pencil strokes parallel to the $x$-axis, shading right on top of the negative $z$-axis, and not shading on top of the positive $z$-axis. These moves make the $xy$-plane look “flat” and seem to pass “behind” the positive $z$-axis.]

The point of Exercise 1 is that we ought to cultivate our three-dimensional drawing capabilities in order to better visualize the analytical mathematics. This is certainly the name of the game in Calculus IV; no time like the present!

**Exercise 2** Consider the point $P = (2, 3, 0)$ in the $xy$-plane. Now pick any other point $Q = (a, b, 0)$ in the $xy$-plane and let $\vec{v} = \overrightarrow{PQ}$. Calculate $\vec{k} \cdot \vec{v}$.

**Exercise 3** Now pick any other point $Q = (a, b, c)$ not in the $xy$-plane and let $\vec{v} = \overrightarrow{PQ}$. Calculate $\vec{k} \cdot \vec{v}$.

**Exercise 4** Argue that the $xy$-plane can be characterized as the set of all points $(x, y, z)$ such that

$$\langle x - 2, y - 3, z - 0 \rangle \cdot \langle 0, 0, 1 \rangle = 0.$$  

The whole construction of exercises 1 through 4 can be repeated for any point in any plane, not just the point $P = (2, 3, 0)$ in the $xy$-plane. Moreover, there is nothing special about taking the dot product of $\overrightarrow{PQ}$ with $(0, 0, 1)$ specifically.

**Exercise 5** Sketch the plane consisting of all points $(x, y, z)$ such that

$$\langle x + 2, y - 0, z - 0 \rangle \cdot \langle 1, 0, 1 \rangle = 0.$$  

**Exercise 6** Sketch the plane consisting of all points $(x, y, z)$ such that

$$\langle x - 2, y - 2, z - 4 \rangle \cdot \langle 2, -1, 3 \rangle = 0.$$  

**Exercise 7** By expanding the dot product condition, show that the plane of exercise 6 is the set of all points $(x, y, z)$ such that

$$2x - y + 3z = 14.$$
The first seven exercises share a common thread. Namely, each plane is uniquely determined by a known point $P$ in the plane and a perpendicular normal vector $\vec{n}$ that determines the “tilt” of the plane. In exercise 4, we characterized the $xy$-plane as the unique plane through the point $P = (2, 3, 0)$ with normal vector $\vec{n} = (0, 0, 1)$.

**Exercise 8** Identify who is playing the role of $P$ and who is playing the role of the normal vector $\vec{n}$ in each of the exercises 5 and 6.

**Exercise 9** Looking back now at exercise 7, is it easy to pick out a normal vector $\vec{n}$ for the plane $2x - y + 3z = 14$? Can you generalize this observation into a theorem?

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**Discussion Questions**

1. Must two planes in $\mathbb{R}^3$ intersect? If not, how can you tell at a glance of their equations whether they do or do not intersect?

2. When two planes intersect, what geometrical object is the set of points $(x, y, z)$ common to both planes?

3. How would you find where (or if) the $z$-axis pierces the plane $31x - 70y + \pi z = 12$?

4. How many points determine a line in $\mathbb{R}^2$? What is the analogous statement concerning planes in $\mathbb{R}^3$?

5. Given two vectors in a plane, how could you create a normal vector $\vec{n}$ for that plane?

And finally, the biggest question of all...

If calculus I is all about whether a curve has a tangent line, then calculus IV is all about whether a [??????????] has a tangent [?????????]! (You may now savor the giddy anticipation of next quarter and have a lovely weekend!)