Solutions to Review Problems on Derivatives

Note that the derivatives may not be fully simplified. 1. Find the derivative $\frac{dy}{dx}(= f'(x))$:

a) $y' = f'(x) = \cos(\cos(\tan x)) \cdot (-\sin(\tan x)) \cdot \sec^2 x$

b) $y' = f'(x) = 12x\sec^6(x^2+1)\tan(x^2+1)$

c) $y' = f'(x) = \frac{(x^3+4x-9)\cos x-(3x^2+4)\sin x}{(x^3+4x-9)^2}$

d) $y' = f'(x) = -\frac{2(\tan(x^2-3)+2x^2\sec^2(x^2-3))}{(x\tan(x^2-3))^3}$

e) $y' = f'(x) = (\sin 1)x^{(\sin 1-1)}$

f) $y' = f'(x) = 2x$

g) $y' = f'(x) = \frac{1-2xy^2}{2x^2y+\cos y}$

2. a)The slope of the line segment L is $\frac{f(b)-f(a)}{b-a}$.

b) Geometrically, this says that the tangent line at the point $(c, f(c))$ is parallel to L.

c) The right hand side represents your *average* velocity over the time interval $a \leq t \leq b$. The left hand side represents your *instantaneous* velocity at time $t = c$.

d) The Mean Value Theorem guarantees that you went 100 km/hr at some point on your trip (maybe at many instants along the way, but definitely at at least one instant).

3. a) $f(x)$ is increasing where $f'(x) > 0$, i.e when $x > 1$. Similarly, $f(x)$ is decreasing where $f'(x) < 0$, i.e. when $x < 1$.

b) Based on part a) there is a local minimum at $x = 1$. There is no local maximum.

c) $f'(x)$ is increasing where $f''(x) > 0$ and $f'(x)$ is decreasing where $f''(x) < 0$. Computing the second derivative, one obtains

$$f''(x) = (x-2)(3x-4)$$

Thus, $f'(x)$ is decreasing on the interval $(4/3, 2)$ and $f'(x)$ is increasing on $(-\infty, 4/3) \cup (2, \infty)$.

d) Concavity is determined precisely by the sign of the second derivative. Thus, by part c), $f(x)$ is concave up on $(-\infty, 4/3) \cup (2, \infty)$ and concave down on $(4/3, 2)$.

e)An inflection point is where the concavity changes. By part d) this happens when $x = 4/3$ and $x = 2$.