There are four problems. Do them all. You have choices in problems 3 and 4. Neatness counts. Box your answers.

<table>
<thead>
<tr>
<th>Lab</th>
<th>Day 1</th>
<th>Final</th>
<th>Total</th>
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1. a) Circle T for true or F for false.
   i. The point \( x_0 = 1 \) is a regular singular point of the Legendre equation
   \[ (1 - x^2)y'' - 2xy' + 6y = 0. \]
   \[ T \ F \]

   ii. The exponent for a Frobenius series solution to the equation \( xy'' + 3y' + xy = 0 \) is \( r = 0 \).
   \[ T \ F \]

   iii. The recursion relation for a power series solution to \( y' + xy = 0 \) is \( a_n = -\frac{a_{n-1}}{n+1} \).
   \[ T \ F \]

   iv. The Laplace transform of \( f(t) = u(t - 2)t^2 \) is \( F(s) = e^{-2s} \cdot \frac{2}{s^3} \).
   \[ T \ F \]

   v. The inverse Laplace transform of \( F(s) = \frac{e^{-\pi s}}{s^2 + 4} \) is \( u(t - \pi) \sin(2t) \).
   \[ T \ F \]

b) Fill in the blanks.
   i. The general solution to \( xy'' + 3y' + xy = 0 \), in terms of Bessel functions, is
   \[ y(x) = \]
   \[ \] .

   ii. The Laplace transform of \( t \cos(2t) \) is
   \[ \] .

   iii. We showed in class yesterday that \( J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin(x) \). It is also true that \( J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos(x) \).
   Use these facts, and the recursion relation for the Bessel functions, to obtain a formula for \( J_{3/2}(x) \) in terms of sines and cosines. Show your work in the space below.
   \[ J_{3/2}(x) = \]
   \[ \] .
2. Obtain the solution to the initial value problem 
\[ y'' + 4y = f(t), \quad y(0) = 1, \quad y'(0) = 0, \] where the driver \( f \) is defined as follows.

\[
f(t) = \begin{cases} 
0 & , \quad 0 < t < \pi \\
1 & , \quad \pi < t < 3\pi \\
0 & , \quad 3\pi < t
\end{cases}
\]

See the graph.

Add a sketch of the solution curve to the plot on the right.
3. Do part a or b, not both.

a) Use the method of Laplace transforms to solve the initial value problem

\[ x'' + 3x' + 2x = \delta(t - 2), \quad x(0) = 1, \quad x'(0) = -2. \]

b) Use Laplace transforms to obtain the solution to the following system of differential equations and initial conditions.

\[
\begin{align*}
x' &= x - y, \quad x(0) = 1 \\
y' &= x + y, \quad y(0) = 0
\end{align*}
\]
4. Do part a or part b, not both.

a) Find the recursion relation for a power series solution for \( y'' + x^2 y' + 2xy = 0 \). Use the recursion relation to write out the first four non-zero terms of the power series solution that satisfies the initial conditions \( y(0) = 0, y'(0) = 1 \).

b) Consider the differential equation \( x^2 y'' + (x^2 - 2)y = 0 \).

   i. Obtain the exponents of the possible Frobenius series solutions expanded about \( x = 0 \).

   ii. Use what you found in part i to write out the form of two linearly independent solutions valid for \( x > 0 \).

      \[
      y_1 = \ldots \\
      y_2 = \ldots \\
      \]

   iii. Obtain one Frobenius series solution to this equation, valid for \( x > 0 \). Write the solution in summation notation.

   iv. Show that the second solution is not a Frobenius series.