Math 241 (Camp)

Triple Integral Worksheet

Let

$$I = \int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z)\ dz\, dy\, dx$$

1. Define E algebraically and then find and sketch the 6 potential bounding surfaces for the region of integration E. *Generally, all 6 surfaces will not actually appear as sides of the solid; this is particularly the case for the surfaces defined by the outer variable.*

2. Sketch E. *If you find this difficult, start the next step: finding the 3 2D-projections. Often, it takes going back-and-forth between the 3D-solid and the 2D-projections to get the full picture.*

3. Find and sketch the 3 projections of E onto the coordinate planes: x-y, x-z, y-z.
   Find and label all curves for each 2D region.
   What are the bounds for the inner variable for each of these projections?
   Remember, the edge curves of the solid are the intersections of two of the bounding surfaces of the solid. You might find it easiest to start with the projection in z onto the x-y plane, since that's the order of the given integral.

4. Rewrite in integral as an equivalent iterated integral in the 5 other orders of integration. *Again, you might find it easiest to start with the projection in z onto the x-y plane but reverse the order if integration between x and y, i.e., $dV = dz\,dx\,dy$. Then do both choices for each of the other projections.*