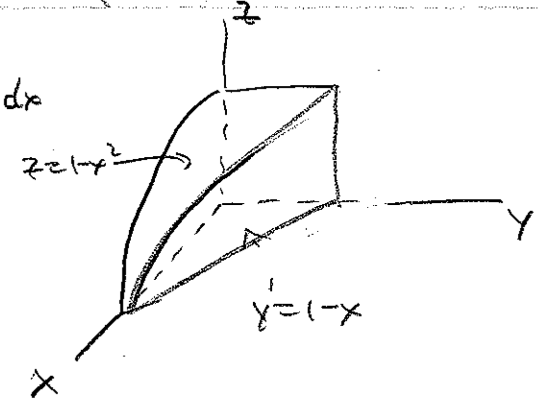
32) $I = \int_0^1 \int_0^{1-x^2} \int_0^{1-x} f(x,y,z)\, dy\, dz\, dx$

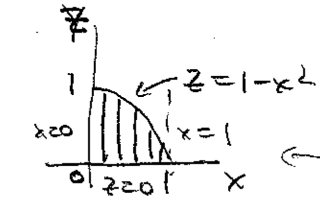


a) D is projection into x-z plane

$\boxed{dy\,dz\,dx}$ • as given:

$0 \le x \le 1$ outer
$0 \le z \le 1-x^2$ middle } D
$0 \le y \le 1-x$ inner

so $D: \{(x,z) \mid 0 \le x \le 1,\ 0 \le z \le 1-x^2\}$

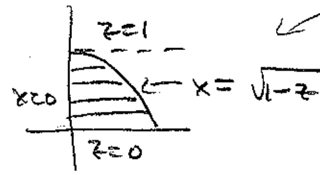


lines parallel to y-axis define solid for both of these,

$\boxed{dy\,dx\,dz}$ • keep y as innermost integral, exchange order in D to get

$D: \begin{cases} 0 \le z \le 1 \\ 0 \le x \le \sqrt{1-z} \end{cases}$

$0 \le y \le 1-x$

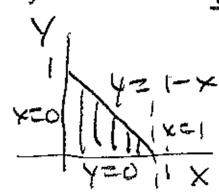


$\Rightarrow I = \int_0^1 \int_0^{\sqrt{1-z}} \int_0^{1-x} f(x,y,z)\, dy\, dx\, dz$

b) D is projection into x-y plane

$\boxed{dz\,dy\,dx}$ • in original, exchange order of y & z integrals; i.e. do z first, x last

$\Rightarrow \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1-x \\ 0 \le z \le 1-x^2 \end{cases}$ } D



lines parallel to z-axis now define the solid

$I = \int_0^1 \int_0^{1-x} \int_0^{1-x^2} f(x,y,z)\, dz\, dy\, dx$

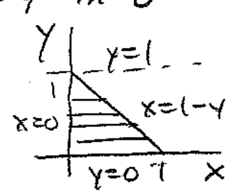
(note w/ x as outer, bounds of y & z are independent of each other ⇒ we can simply exchange their order)

$\boxed{dz\,dx\,dy}$ • keeping z as innermost, exchange order of x-y in D

here too

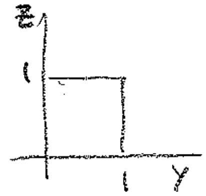
$D: \begin{cases} 0 \le y \le 1 \\ 0 \le x \le 1-y \end{cases}$

$0 \le z \le 1-x^2$



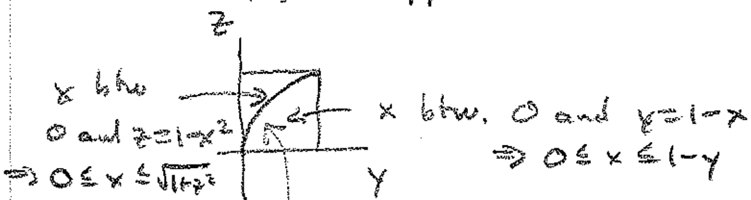
$I = \int_0^1 \int_0^{1-y} \int_0^{1-x^2} f(x,y,z)\, dz\, dx\, dy$

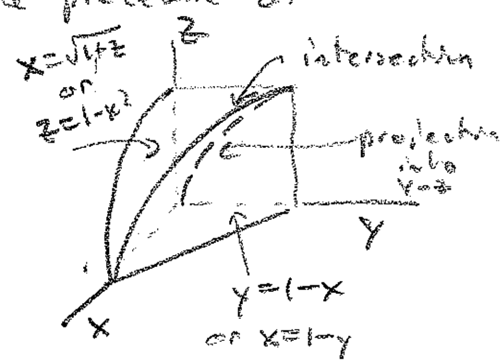
c) We've done both possibilities for y as innermost and for z as innermost integral,
(i.e. D is projection into x-z and x-y planes, respectively)
Now, we'll try it projecting into y-z plane for D, i.e. x as innermost

$\Rightarrow D:$ 

$\Rightarrow \begin{matrix} 0 \le y \le 1 \\ 0 \le z \le 1 \end{matrix}$ and lower bound for x is $x \ge 0$ $\Rightarrow 0 \le x$

unfortunately, the upperbound for x is a piecewise dt^n



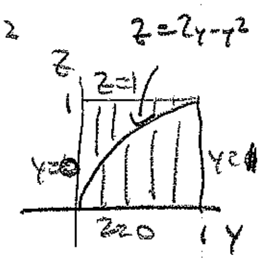


$\Rightarrow$ we need to find the curve in the y-z plane which separates these regions;
it's the projection of the intersection of $z=1-x^2$ and $y=1-x$ into the y-z plane

intersection occurs when $\begin{cases} z=1-x^2 \\ y=1-x \end{cases}$ both true

$y=1-x \Rightarrow x=1-y \therefore z=1-(1-y)^2 \Rightarrow z=2y-y^2$ or $y=1-\sqrt{1-z}$

so D is the unit square in y-z plane divided along $z=2y-y^2$



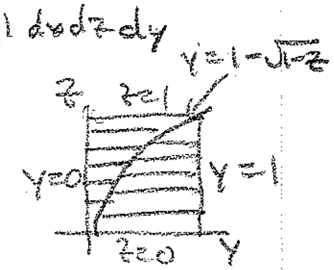
$\boxed{dx\,dz\,dy}$

$$\begin{cases} 0 \le y \le 1, & 0 \le z \le 2y-y^2, \quad 0 \le x \le 1-y \\ \quad '' & 2y-y^2 \le z \le 1, \quad 0 \le x \le \sqrt{1+z} \end{cases}$$

$$\Rightarrow I = \int_0^1 \int_0^{2y-y^2} \int_0^{1-y} f(x,y,z)\,dx\,dz\,dy + \int_0^1 \int_{2y-y^2}^1 \int_0^{\sqrt{1+z}} f(x,y,z)\,dx\,dz\,dy$$

$\boxed{dx\,dy\,dz}$

- swapping order of z & y



$$\begin{cases} 0 \le z \le 1, & 0 \le y \le 1-\sqrt{1-z}, \quad 0 \le x \le \sqrt{1+z} \\ \quad '' & 1-\sqrt{1-z} \le y \le 1, \quad 0 \le x \le 1-y \end{cases}$$

$$I = \int_0^1 \int_0^{1-\sqrt{1-z}} \int_0^{\sqrt{1+z}} f(x,y,z)\,dx\,dy\,dz + \int_0^1 \int_{1-\sqrt{1-z}}^1 \int_0^{1-y} f(x,y,z)\,dx\,dy\,dz$$

1.