1. Let $f : A \to B$ and $D \subseteq B$. Prove that $f(f^{-1}(D)) = D \cap \text{Range } f$.

2. Let $f : A \to B$ and $C, D \subseteq A$. Prove that $f(D \cup C) = f(D) \cup f(C)$.

3. Let $f : A \to B$ and $C, D \subseteq B$. Prove that $f^{-1}(D \cup C) = f^{-1}(D) \cup f^{-1}(C)$.

   Given $f : A \to B$ we define the induced functions $\overline{f} : \mathcal{P}(A) \to \mathcal{P}(B)$ by $\overline{f}(C) = f(C)$ for all $C \in \mathcal{P}(A)$ and $\hat{f} : \mathcal{P}(B) \to \mathcal{P}(A)$ by $\hat{f}(D) = f^{-1}(D)$ for all $D \in \mathcal{P}(B)$.

4. Let $f : A \to B$. Prove that if $f$ is injective then $\overline{f}$ is injective.

5. What condition will make the induced function $\hat{f} : \mathcal{P}(B) \to \mathcal{P}(A)$ injective?

6. Let $A$ and $B$ be sets. If $A$ is finite, prove that $A \cap B$ is finite.

7. Let $A$ and $B$ be sets. If $A$ and $B$ are finite, prove that $A \cup B$ is finite.

8. Let $A$ and $B$ be sets such that $A \subseteq B$. If $A$ is infinite, prove that $B$ is infinite.

9. Let $f : A \to B$ be surjective and $A$ be finite. Prove that $B$ is finite.

The grader will carefully consider 7 and 9.