Read sections:
1. 1.6 (for 1/17)

Do the following problems:
1. 1.5.2: Determine whether the following sets are linearly dependent or linearly independent.
   (a) \( \left\{ \begin{pmatrix} 1 & -3 \\ -2 & 4 \end{pmatrix}, \begin{pmatrix} -2 & 6 \\ 4 & -8 \end{pmatrix} \right\} \) in \( M_{2 \times 2}(\mathbb{R}) \).
   (b) \( \left\{ \begin{pmatrix} 1 & -2 \\ -1 & 4 \end{pmatrix}, \begin{pmatrix} -1 & 1 \\ 2 & -4 \end{pmatrix} \right\} \) in \( M_{2 \times 2}(\mathbb{R}) \).
   (c) \( \{x^3 + 2x^2, -x^2 + 3x + 1, x^3 - x^2 + 2x - 1\} \) in \( P_3(\mathbb{R}) \).
   (d) \( \{x^3 - x, 2x^2 + 4, -2x^3 + 3x^2 + 2x + 6\} \) in \( P_3(\mathbb{R}) \).
   (e) \( \{(1, -1, 2), (1, -2, 1), (1, 1, 4)\} \) in \( \mathbb{R}^3 \).
   (f) \( \{(1, -1, 2), (2, 0, 1), (-1, 2, -1)\} \) in \( \mathbb{R}^3 \).

2. 1.5.9: Let \( u \) and \( v \) be distinct vectors in a vector space \( V \). Show that \( \{u, v\} \) is linearly dependent if and only if \( u \) or \( v \) is a multiple of the other.

3. 1.5.10: Give an example of three linearly dependent vectors in \( \mathbb{R}^3 \) such that none of the three is a multiple of another.

4. 1.5.13: Let \( V \) be a vector space over a field of characteristic not equal to two (so just think \( \mathbb{R} \)).
   (a) Let \( u \) and \( v \) be distinct vectors in \( V \). Prove that \( \{u, v\} \) is linearly independent if and only if \( \{u+v, u-v\} \) is linearly independent.
   (b) Let \( u, v, \) and \( w \) be distinct vectors in \( V \). Prove that \( \{u, v, w\} \) is linearly independent if and only if \( \{u+v, u+w, v+w\} \) is linearly independent.

5. 1.5.16: Prove that a set \( S \) of vectors is linearly independent if and only if each finite subset of \( S \) is linearly independent.

6. 1.6.7: The vectors \( u_1 = (2, -3, 1), u_2 = (1, 4, -2), u_3 = (-8, 12, -4), u_4 = (1, 37, -17), \) and \( u_5 = (-3, -5, 8) \) generate \( \mathbb{R}^3 \). Find a subset of the set \( \{u_1, u_2, u_3, u_4, u_5\} \) that is a basis for \( \mathbb{R}^3 \).

7. 1.6.9: The vectors \( u_1 = (1, 1, 1, 1), u_2 = (0, 1, 1, 1), u_3 = (0, 0, 1, 1), \) and \( u_4 = (0, 0, 0, 1) \) form a basis for \( P_4 \). Find the unique representation of an arbitrary vector \( (a_1, a_2, a_3, a_4) \) in \( P_4 \) as a linear combination of \( u_1, u_2, u_3, \) and \( u_4 \).

8. 1.6.11: Let \( u \) and \( v \) be distinct vectors of a vector space \( V \). Show that if \( \{u, v\} \) is a basis for \( V \) and \( a \) and \( b \) are nonzero scalars, then both \( \{u + v, au\} \) and \( \{au, bv\} \) are also bases for \( V \).

9. 1.6.20: Let \( V \) be a vector space having dimension \( n \), and let \( S \) be a subset of \( V \) that generates \( V \).
   (a) Prove that there is a subset of \( S \) that is basis for \( V \). (Be careful not to assume that \( S \) is finite).
   (b) Prove that \( S \) contains at least \( n \) vectors.

The grader will carefully consider 1.5.16 and 1.6.20 so you should write these up more carefully.