1. Prove that $\mathbb{Z}_2 \oplus G$ is not simple where $G$ is any group.

2. Let $G$ be a group with binary operation $\ast$. Prove that $\ast$-inverses are unique (i.e., if $a, b \in G$ such that $a \ast b = e$, then $b = a^{-1}$).

3. Write out the group elements and the multiplication table for $A_4$. (There are 12 elements; make up some notation).

4. Prove that any group with even order has an element of order 2 (i.e., there is an element $a$ such that $a \neq e$ and $a^2 = e$).

5. Prove that $\mathbb{Z}_p$ is simple when $p$ is prime. Try to avoid using Lagrange’s theorem.