Math 414, Analysis, Spring 2007

Problem Set 5, due Friday, June 8

1. Prove the following: if \( P \, dx + Q \, dy \) is an exact differential, and \( C \) is a piecewise smooth curve from \( p \) to \( q \), then there is a function \( f \) such that \( \int_C P \, dx + Q \, dy = f(q) - f(p) \).

2. Show that \( P \, dx + Q \, dy \) is an exact differential on the unit disk in \( \mathbb{R}^2 \) if \( P = y^2 e^{xy} \) and \( Q = (1 + xy) e^{xy} \).

3. Is \( P \, dx + Q \, dy \) an exact differential on the punctured open unit disk in \( \mathbb{R}^2 \) (\( \{ (x, y) \mid 0 < x^2 + y^2 < 1 \} \)) if \( P = \frac{y}{x^2 + y^2} \) and \( Q = \frac{x}{x^2 + y^2} \)? What about on the open rectangle \( (0, 1) \times (0, 1) \)?

4. If the functions \( P, Q, P_2, \) and \( Q_1 \) are continuous in \( D \) and \( P \, dx + Q \, dy \) is an exact differential in \( D \), then prove that \( P_2 = Q_1 \) throughout \( D \).

5. Suppose that \( S \) is an elementary set

\[
S = \{(x, y) \mid a \leq x \leq b, \mu(x) \leq y \leq v(x)\}
\]

and \( f \) is a continuous function on \( S \). Prove that

\[
\int_S f(x, y) \, dA = \int_a^b \int_{\mu(x)}^{v(x)} f(x, y) \, dy \, dx.
\]