Math 414, Analysis, Spring 2007

Problem Set 4, due Friday, May 25

1. Suppose that $f$ is a function on $\mathbb{R}^2$ such that $f_1$ and $f_2$ are bounded throughout some open ball $B(x, r)$. Prove that $f$ is continuous at $x$. Hint: recall that If $f$ is a function such that its partials exists in $B(x, r)$ and $h \in B(0, r)$, there there are points $p^{(1)}, \ldots, p^{(n)}$ in $B(x, r)$ such that

$$f(x + h) - f(x) = f_1(p^{(1)})h_1 + \cdots + f_n(p^{(n)})h_n.$$  

2. Suppose that the function $f$ has a relative extreme point at $x \in D^o \subset \mathbb{R}^n$ and each of its partial derivative exist in $D^o$. Prove that $f_i(x) = 0$ for all $i$ and $D_uf(x) = 0$ for all unit vectors $u$.

3. Suppose $S$ is a nonempty open set. Prove that $A^-(S) > 0$.

4. Let $R$ be a rectangle and $\mathcal{N}$ be a net on $R$; find $\beta[\mathcal{N}]$.

5. Suppose that $f$ is integrable over the bounded set $S$ and $k \in \mathbb{R}$. Prove that

$$\int\int_S kf(x, y) \, dA = k \int\int_S f(x, y) \, dA.$$