

Name: _____

Section: _____

Calculus IV
Math 241 Fall 2003
Professor Ben Richert
Exam 2
November 13, 2003

Please do all your work in this booklet and show all the steps.
Calculators and note-cards are not allowed.

Problem	Possible points	Score
1	10	
2	10	
3	10	
4	30	
5	10	
Total	70	

2

Problem 1. (10 pts.) A 15-ft-by-15-ft hole has been excavated in Cal Poly canyon. You take depth measurements every 5 ft (starting from the southwest corner of the hole), and come up with the following table.

	0	5	10	15
0	3	4	5	3
5	4	4	5	4
10	3	2	3	2
15	5	6	7	6

The next day the hole fills with water during a torrential rain. Estimate the amount of water in the hole using $m = n = 3$ subdivisions (you don't need to add it all up, just write a sum which gives the estimate).

Problem 2. (10 pts.) Compute the surface area of the function $f(x, y) = \frac{x^2}{2} + \frac{y^2}{2}$ above the region between the two circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Problem 3. (10 pts.) Let X be the number of minutes that your math professor spends telling jokes during a 50 minute lecture, and let Y be the number of minutes he spends erasing the chalkboard. Suppose that the joint probability density function for X and Y is given by

$$f(X, Y) = \begin{cases} CXY & \text{if } 0 \leq X \leq 50, 0 \leq Y \leq 50 \\ 0 & \text{otherwise,} \end{cases}$$

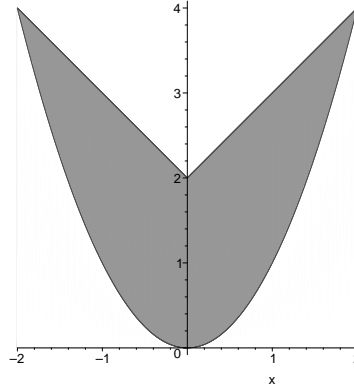
where C is a constant.

(a – 5 pts) Describe the process you would use to find the value of the constant C . (A sentence or two will suffice).

(b – 5 pts) Set up an integral which computes the probability that your professor spends more than 10 minutes telling jokes, and less than 2 minutes erasing the board.

Problem 4. (30 pts.) Do any three of the following (and denote clearly which three those are):

(a - 10 pts.) Let D be the lamina bounded by $y = x^2$ and $y = \begin{cases} x + 2 & \text{if } x \geq 0 \\ -x + 2 & \text{otherwise.} \end{cases}$ and suppose that the density at the point (x, y) is given by the function $\rho(x, y) = 4$.



Set up integrals to compute the center of mass of the lamina.

(b - 10 pts.) Let $f(x, y, z) = \frac{x}{y} e^{x^2 + y^2 + z^2}$ and let E be the region inside the sphere $x^2 + y^2 + z^2 = 4$, above the xy -plane, and below the cone $z = \sqrt{\frac{x^2 + y^2}{3}}$. Set up the integral $\iiint_E f(x, y, z) dV$ using spherical coordinates.

	10
	10

(c - 10 pts.) Consider the integral $\int_0^1 \int_0^{1-x^2} \int_0^{1-x} (x+y+1) dz dx dy$. Rewrite this integral as $\iiint_E (x+y+1) dy dx dz$ (i.e., find the appropriate limits of integration).

(d - 10 pts.) Let E be the region consisting of the intersection of the sphere of radius 2 (centered at the origin) with the cylinder $(x-1)^2 + y^2 = 1$. Set up an integral to compute the volume of E .

	10
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(e - 10 pts.) Let $f(x, y, z) = \sqrt{x^2 + 1}\sqrt{y^2 + 1}$ and let E be the region in the first octant bounded by the planes $z = x$ and $y = 1$, and the curve $x = \sqrt{y}$. Set up an integral to compute $\iiint_E f(x, y, z) \, dV$.

(f - 10 pts.) Suppose that E is the region bounded by the ellipsoid $\frac{x^2}{9} + \frac{y^2}{4} + z^2 = 1$. Set up an integral to evaluate $\iiint_E x \, dV$ using the transformation $x = 3u$, $y = 2v$, and $z = c$.

	10
	10

Problem 5. (10 pts.)

Match the following functions with the plots of their gradient vector fields and their level curves.

Function	Plot of Gradient Vector Field	Level Curve
(a) $f(x, y) = x^2y + y$		
(b) $f(x, y) = \sin(x) + y$		
(c) $f(x, y) = \ln(x^2 + y^2 + 1)$		
(d) $f(x, y) = \sin(y) + x$		
(e) $f(x, y) = x^2 + y^2$		

