Name: _____

Section: _____

Calculus IV

Math 241 Fall 2003

Professor Ben Richert

Exam 1

October 16, 2003

Please do all your work in this booklet and show all the steps.
Calculators and note-cards are not allowed.

Problem	Possible points	Score
1	15	
2	10	
3	15	
4	10	
5	15	
6	5	
7	10	
8	10	
Total	90	

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Problem 1. *(15 pts.)*

(a – 5 pts) *Find the gradient of the function* $f(x, y) = 16x + e^{x \sin(xy)}$.

(b – 5 pts) *Find the maximum rate of change of* $f(x, y)$ *at the point* $(2, 0)$.

(c – 5 pts) *Find the rate of change of* $f(x, y)$ *at the point* $(2, 0)$ *in the direction of the vector* $\overline{\mathbf{v}} = \langle 1, 2 \rangle$.

Problem 2. *(10 pts.)*

a – 5 pts. *Show that*

$$\lim_{(x,y)\to(0,0)} \frac{2x^2+3xy+4y^2}{3x^2+5y^2}$$

does not exist.

b – 5 pts. *Is*

$$f(x,y)=\begin{cases}\dfrac{2x^2+3xy+4y^2}{3x^2+5y^2} & \text{for } (x,y)\neq(0,0)\\ 0 & \text{for } (x,y)=(0,0)\end{cases}$$

continuous at $(0,0)$*?*

[4]
Problem 3. *(15 pts.)*

(a – 5 pts) *Suppose that the function* $p(x, y) = x^2 + y^2 + 6x - 3y + 5$ *gives the profit (in millions) made by manufacturing* x *buttons per year (in millions) and* y *strips of velcro per year (again in millions). This year you manufactured* 1 *(million) buttons and* 2 *(million) velcro strips. How much profit did you make?*

(b – 5 pts) *Find the tangent plane to the graph of* f *at the point* $(1, 2, f(1, 2))$.

(c – 5 pts) *Suppose that due to an inability to add, you may have underestimated your button production by at most* 0.1 *and underestimated your velcro production by at most* 0.2 *(both in millions). Use differentials to estimate the maximum possible error in your profit calculation.*

Problem 4. *(10 pts.) Consider the following table of values of the function $f(x, y)$:*

	1	2	3
2	3	4	6
3	4	4	3
4	3	2	1

Use these values to estimate $f_{xy}(3, 2)$.

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Problem 5. *(15 pts.) Find all the critical points of the function* $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$ *and classify them as maximum, minimum, or saddle points.*

Problem 6. *(5 pts.) Describe the intersection of a solid vertical cylinder of radius* 1 *(centered at the origin) and the solid sphere of radius* 2 *(again centered at the origin) in rectangular coordinates.*

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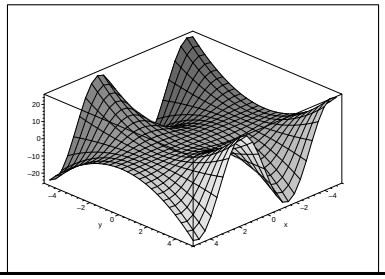
Problem 7. *(10 pts.) The voltage V in a simple electrical circuit is slowly decreasing as the battery wears out. The resistance R is slowly increasing as the resistor heats up. Use Ohm's Law, $V = IR$, to find how the current I is changing at the moment when $R = 400\ \Omega$, $I = 0.08\ A$, $dV/dt = -0.01\ V/s$, and $dR/dt = 0.03\ \Omega/s$.*

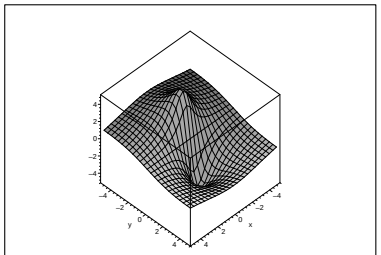
Problem 8. *(10 pts.)*
Match the following functions, graphs, level curves by filling up the table below:

Function	***Graph***	***Level Curves***
(a) $f(x,y) = x^3y^3e^{-x^2-y^2}$		
(b) $f(x,y) = \dfrac{-10y}{x^2+y^2+1}$		
(c) $f(x,y) = y^2\sin(x)$		
(d) $f(x,y) = e^{-x^2} + e^{-y^2}$		
(e) $f(x,y) = \cos(x)^2 + y$		

(I)



(II)



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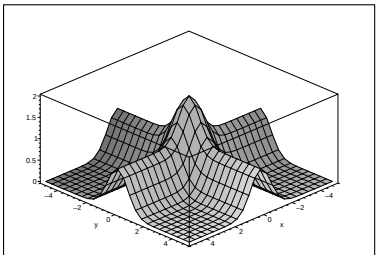

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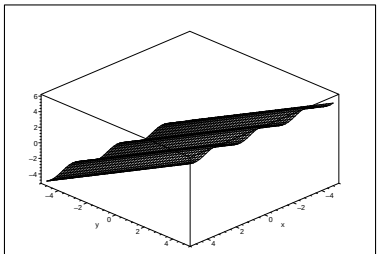

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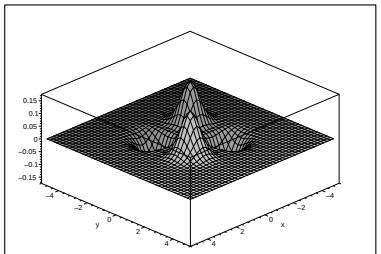
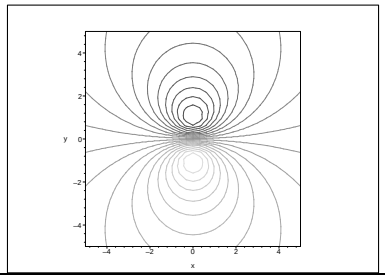


(A)



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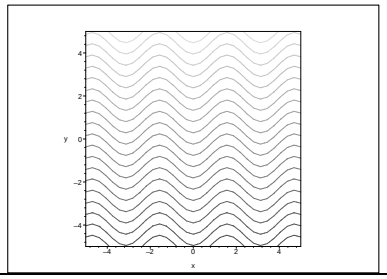
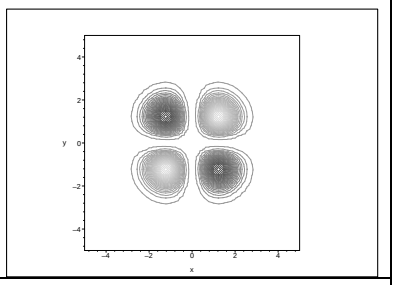
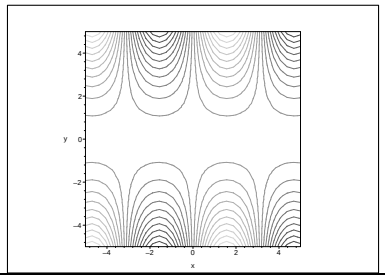


(C)



(D)



(E)