1. Do problems: 3.4.9bce, 3.4.12, 3.4.18, 3.4.19

2. One way of designing a tennis tournament is to have each entered player compete against every other player exactly once (with no ties allowed). We will abuse notation slightly and refer to a tournament as the outcome of such an event (so if we were to say, just by way of a made up example, “suppose there was a tournament in which no player lost every match” we mean to indicate that there was a contest, all the matches have been played, and it turned out that every entrant won at least one match). Given a tournament, a player \( x \) is said to be a top player if for every other player \( y \), either \( x \) beat \( y \), or there is a third player \( w \) such that \( x \) beat \( w \) and \( w \) beat \( y \).

(a) Show that a tournament can have more than one top player.

(b) Show that every tournament always has at least one top player. (Hint: show that for all \( n \geq 2 \) every tournament consisting of \( n \) players will have at least one top player).

The grader will grade 3.4.9c and 3.4.18, so you should write these up more carefully.