Math 248, Methods of Proof, Fall 2013

Homework 10, due Friday, 11/1

1. Do problems: 3.5.5a, 3.5.8, 3.5.13

2. Let $F_i$ be the $i$th Fibonacci number (so $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-1} + F_{n-2}$ for all $n \geq 3$). Prove that given $a \in \mathbb{N}$, $F_aF_n + F_{a+1}F_{n+1} = F_{a+n+1}$ for all $n \in \mathbb{N}$.

3. Let $a_1 = 2$, $a_2 = 4$ and $a_{n+2} = 5a_{n+1} - 6a_n$ for all $n \geq 1$. Prove that $a_n = 2^n$.

4. Prove that the Well Ordering Axiom implies Strong Induction. That is, in a math world in which the Well Ordering Axiom holds, prove that Strong Induction holds as well. Start as follows. Let $S \subset \mathbb{N}$ be a set such that $1 \in S$ and for all $n \in \mathbb{N}$, if $\{1, \ldots, n\} \in S$, then $n + 1 \in S$. You need to conclude somehow that $S = \mathbb{N}$. Suppose not. Then we can let $T$ be the set $\mathbb{N} - S$. By hypothesis, $T \neq \emptyset$. So by the Well Ordering Axiom,…

5. Consider the following theorem.

Theorem 1. All babies have the same color eyes.

Proof. Let $S$ be the set of all positive integers $n$ for which any set of $n$ babies has the same color eyes. Clearly $1 \in S$, so in order to prove the theorem it is enough, by induction, to show that if $n \in S$, then $n + 1 \in S$. Let $B$ be a set of $n + 1$ babies for $n \geq 1$, and split $B$ into two subsets $R$ and $L$ each containing $n$ babies and such that $R \cup L = B$ (we can do this, for instance, by ordering the babies 1 to $n + 1$, and then letting $L$ consist of the babies numbered 1 to $n$ and $R$ consist of the babies numbered 2 to $n + 1$). Now each of the babies in $R$ has the same eye color (by the induction hypothesis), and the same is true for each of the babies in $L$. Meanwhile the $n - 1$ babies in $R \cap L$ have the same color eyes as both those in $R$ and $L$ simultaneously. We conclude therefore that all $n + 1$ babies have the same color eyes. Thus by induction, all babies must have the same color eyes. 

Obviously this theorem is false. Find the error in the proof.

The grader will grade 3.5.13 and number 3 above, so you should write these up more carefully.