1. Attempt to read the paper (On the Number of Primes Less Than a Given Magnitude) in which Riemann posits his famous conjecture. The idea, of course, is to understand as much of it as you can, and to write an expository paper about the result. The difficulty will depend; you could, of instance, attempt to only understand his arguments about the zeta function for real inputs, which should make things easier.

2. Riemann’s hypothesis, if true, implies a large number of other theorems (I believe it is something on the order of 500, but don’t quote me on that). Write a paper exploring these. For instance, how are they proved? How important are they? My suspicion is that this should prove to be of medium difficulty—sources for this information exist.

3. The Riemann hypothesis has been checked for $s$ such that $0 < \text{Im}(s) < N$ where $N$ is some very large number. That is to say, it is known that if $s$ is a root of $\xi(s)$ and $\text{Im}(s) < N$, then $\text{Re}(s) = 1/2$. The proof usually consists of somehow counting the number of roots with $\text{Im}(s) < \gamma$, and then finding that many roots with $\text{Im}(s) < \gamma$ and $\text{Re}(s) = 1/2$. The details of this procedure should make for an interesting senior project. What are the relevant theorems? How are they proved? How does the search for zeros proceed? What are the computational issues involved? I believe this is an accessible project.