1. Write an expository paper on the undecidability of the continuum hypothesis (which states that the cardinality of the real numbers is equal to the cardinality of the powerset of the natural numbers). Of course, the uncountability of the reals is a central player in this drama. To keep this project from becoming an uninteresting book report, one would have to be careful to explain and explore the ZFC axioms as well as the meaning of undecidability.

2. The author of the current paper refers us to work by R. Telgársky (Topological games: On the 50th anniversary of the Banach-Mazur game, Rocky Mountain J. Math., 17 (1987) 227-276) for more examples of “games” which can be used to prove interesting theorems, and claims that some of these “games” are not completely understood. A great senior project would be to obtain that paper in order to explore a few of the games, including the relevant background, and then try to understand some of the open questions or perhaps attempt to solve some familiar problem with a “game.” The first part of this shouldn’t be too bad, assuming some of the theorems don’t require too much background. The second part would be a challenge, but we like those.

3. It is well known that the Axiom of Choice (the “C” in ZFC set theory) is independent of the Zermelo-Fraenkel axioms (the ZF, of course). It turns out that you can replace C with the Axiom of Determinacy, and that D (as we will call it) is inconsistent with C. (The Axiom of Determinacy says roughly that one or the other players of a certain game—similar to the real number game—always has a winning strategy, that is, that every possible version of the game is determined). An interesting senior project would be to explore the ZFD set theory (what theorems can be proved, etc). Apparently, ZFD is not considered to be a realistic alternative to ZFC. Why not? This would probably turn out to be difficult, unless one was already interested in set theory.