Calculus II
Math 142 Spring 2007
Professor Ben Richert

Final
June 15, 2007

Please do all your work in this booklet and show all the steps.
Calculators are not allowed. You may use one 3 × 5 notecard.

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<th>Problem</th>
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<td><strong>Total</strong></td>
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A few integrals:

1. \[ \int \sec^3(u) \, du = \frac{1}{2} \tan(u) \sec(u) + \frac{1}{2} \ln |\sec(u) + \tan(u)| + C \]
2. \[ \int \sqrt{1 + u^2} \, du = \frac{u \sqrt{a^2 + u^2}}{2} + \frac{a^2}{2} \ln |x + \sqrt{a^2 + u^2}| + C \]
3. \[ \int \frac{\sqrt{a + bu}}{u} \, du = 2 \sqrt{a + bu} + \sqrt{a} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C \text{ if } a > 0 \]
4. \[ \int u \cos^{-1} u \, du = \frac{2u^2 - 1}{4} \cos^{-1} u - \frac{u \sqrt{1 - u^2}}{4} + C \]
Problem 1 (10pts). Suppose that

\[ f(x) = \frac{t(t + 1)^4(t + 2)^5}{(t + 3)^3(2t^2 + 1)} \]

calculates the total number of calories consumed in your ice-cream shop after you have been opened for \( t \) hours. What is the rate at which calories are being consumed 1 hour after opening? (Hint: you know a special technique for dealing with interesting expressions like this one).
Problem 2 (20pts). An turtle escapes from the biology lab and races across campus. You follow, and make the following observations about his speed $v(t)$ (in meters per minute):

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<tr>
<td>$v(t)$</td>
<td>1</td>
<td>1/2</td>
<td>1/4</td>
<td>1/2</td>
<td>2</td>
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(a–10pts) Estimate the total distance travelled by the turtle during the first 6 minutes of his flight (use the midpoint rule if an integral is necessary).

(b–10pts) Suppose that you know that $v''(t) \leq 1/10$. Give an upper bound for the error in your estimate in part (a).
Problem 3 (10pts). For what values of $r$ does $y = e^{rt}$ satisfy the differential equation $y'' + y' - 6y = 0$?
Problem 4 (10pts). A 10 meter chain with mass 2 kilograms per meter is suspended off a cliff. Compute the work required to lift the chain to the top of the cliff.
Problem 5 (10pts). A thin metal plate in the shape of a triangle with constant density 5 kg/m can be described as sitting in the xy-plane with corners (0, 0), (2, 0), and (2, 4) (for x and y in meters). Give the coordinates of the center of mass of this triangle.
Problem 6 (20pts). Consider the solid obtained by revolving the region bounded by $y = \cos(x)$, $y = 0$, $x = 0$, and $x = \pi/2$ about the $x$-axis.

(a–10pts) Compute the volume of this solid.
(b-10pts) Compute the surface area of this solid.
Problem 7 (10pts). Decide whether \( \int_{1}^{\infty} \frac{2 + e^{-x}}{x} \, dx \) converges or diverges. You do not need to compute the integral.
Problem 8 (20pts). Compute the following two integrals:

(a-10pts) \( \int \frac{1}{\sqrt{3 - 2x - x^2}} \, dx \)
(b-10pts) \[ \int x \sqrt{x + \pi} \, dx \]
Problem 9. (16pts) Each of the entries on the right corresponds with exactly one the left (after some manipulation). Match the corresponding entries. You do not need to show your work.

\[ \ln(\ln x^x) \quad (A) \ 2 \]
\[ \ln(\ln x)^x \quad (B) \ x \ln(\ln x) \]
\[ f''(0) \text{ if } f(x) = \sin^{-1} x \quad (C) \ -\frac{1}{2^{3/2}} \]
\[ f''(1) \text{ if } f(x) = \sinh^{-1} x \quad (D) \ 1/3 \]
\[ \ln x^{\ln x} \quad (E) \ \ln x + \ln(\ln x) \]
\[ e^{\frac{1}{2} \ln 4} \quad (F) \ (\ln x)^2 \]
\[ \cos(\sin^{-1}(\tan \theta)) \text{ if } \sec \theta = \sqrt{17}/3 \quad (G) \ \ln(2 + \sqrt{3}) \]
\[ \cosh^{-1}(2) \quad (H) \ 0 \]