

# Math 412, Analysis, Fall 2006

## Problem Set 3, due Friday, October 13

1. Let  $x > 0$  be real. Prove (completely), that there is  $y \in \mathbb{R}$  such that  $y^2 = x$ .
2. Prove that the union of two cuts is a cut.
3. Let  $C, D$  be cuts with  $C, D > 0$ . Explain why  $\{q \in \mathbb{Q} \mid q < cd \text{ for some } c \in C, d \in D\}$  is not a cut.
4. Prove that multiplication of cuts is associative.
5. Demonstrate that  $\mathbb{R}$  as we have constructed it is the same as  $\mathbb{R}$  as we usually think about it. That is, find a bijection  $\phi$  from the set of cuts to  $\mathbb{R}$  such that  $\phi(C + D) = \phi(C) + \phi(D)$  and  $\phi(CD) = \phi(C)\phi(D)$  for all cuts  $C$  and  $D$ .

By the way: most of the problems in the text are interesting. These are the ones I am collecting.