

The Effects of the Default and Call Risks on Bond Duration

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Abstract

This paper examines the effects of default risk, call risk, and their interactions on bond duration. We find that call risk decreases duration whereas default risk may increase or decrease duration for risky bonds. But the joint effect of default and call risks always results in shorter duration for corporate bonds, suggesting that the call effect dominates the default effect. Controlling for the effect of default risk, the call risk has a significant negative marginal effect on duration but this effect diminishes as bond rating declines. Controlling for the effect of call risk, default risk generally decreases duration but this effect tends to reverse as the rating declines and maturity increases. These findings are generally consistent with the prediction of Acharya and Carpenter (2002). Finally, the both joint and marginal effects of call and default risks on duration magnify during the recession period with declining interest rates.

I. Introduction

The duration measure is commonly used to hedge against interest rate risk. An accurate duration measure is therefore essential for bond portfolio managers to formulate an effective hedging strategy against any unanticipated interest rate movement. In practice, the Macaulay duration is the most popular measure employed by practitioners to perform risk management. However, prior studies have shown that the Macaulay duration is not a valid measure of bond duration in the presence of default and call risks.

Chance (1990) employs the option-based model to derive the duration of defaultable zero-coupon bonds. Under the assumption that the value of firm assets is independent of interest rates, he finds that the default risk-adjusted duration is always shorter than the Macaulay duration. Nawalkha (1996) extends Chance's work by considering the dependence between the value of firm assets and interest rates. His study shows that the duration of defaultable zero-coupon bonds can be shorter than, equal to, or longer than the Macaulay duration. Xie, Liu and Wu (2005) apply a reduced-form model to examine the interactive effect of default and interest rate risk on the duration of defaultable zero-coupon bonds. They find that the duration for a defaultable bond can be longer than, equal to or shorter than a default-free counterpart depending upon the relation between default intensity and interest rates. Fons (1990) estimates the empirical duration of corporate bond indices and finds that during the 1980-1988 period, the duration of corporate bond indices is always less than the Macaulay duration and the difference between these two duration measures widens as the bond rating declines. However, Duffee (1998) points out that US corporate bond indices are composed of both callable and noncallable bonds. Therefore, Fons' finding for the effect of default risk on duration may be tainted by the call effect.

These studies focus exclusively on the default effect on bond duration. In reality, many defaultable bonds also carry a call provision (see Duffie and Singleton, 1999). Therefore, how call risk and the interaction between call and default risks affect duration is of great interest to financial researchers and practitioners.

In an important paper, Acharya and Carpenter (2002) examine the effects of default and call risks on duration using an option-based valuation model for coupon-bearing callable defaultable bonds. They show that default or call risk alone reduces bond duration. However, when both risks are present, call risk may increase the duration of a defaultable bond, and default risk may increase the duration of a callable bond because the exercise of one option rules out the exercise of the other option. Their simulation shows that when both default and call risks are considered, the effect of the call provision on bond duration diminishes as the probability of default increases, resulting in a longer duration for riskier callable bonds. While Acharya and Carpenter's analysis is insightful, no empirical evidence has been provided to validate their theoretical model.

Jacoby and Roberts (2003) develop a unified framework to examine the impact of both default and call risks on the duration of risky callable bonds. They use a database of Canadian corporate bond indices from 1986 to 1997 to perform empirical tests and find that call risk always shortens duration, but default risk may either lengthen or shorten it. However, they do not directly test the interactive effect of call and default risks on bond duration. Sarkar and Hong (2004) use an option-based valuation model to derive the duration of callable defaultable bonds. Their empirical results show that the call provision generally shortens duration. They find that call risk adjustment for duration is more important for high-grade bonds, whereas default risk adjustment is more important for low-grade bonds. The call feature

lengthens the duration for low-grade bonds but shortens the duration for high-grade bonds. While Sarkar and Hong provide a simple theoretical framework for examining the effects of call and default risks on duration, the predictability of their model may be limited by the stringent assumptions of constant interest rate and perpetual bonds.

Although previous studies have underscored the importance of adjusting the Macaulay duration for the effects of default and call risks, none of them has thoroughly investigated the joint and marginal effects of these factors on duration. In particular, how the interaction between default and call risks affects duration, especially during the period of economic downturn and declining interest rate, remains unclear. It is known that during the economic downturn, issuers of defaultable bonds are prone to exercise the option on default. Also, during the period of declining interest rate, issuers of callable bonds are more likely to exercise the call option. Therefore, call and default risk factors may play a distinct role in affecting duration at different times.

In this paper, we provide empirical evidence on the effects of default risk, call risk, and the interaction between these risks on bond duration. Our empirical study is motivated by the theoretical work of Acharya and Carpenter (2002). To test the implications for their theoretical model, we examine marginal and joint effects of the default and call risks and how these effects vary cyclically. Our results show that the call-adjusted duration is always shorter than the Macaulay duration. By contrast, default risk may increase or decrease bond duration. We find that the default-adjusted duration of low-grade bonds is shorter than the Macaulay counterpart but that of high-grade bonds is longer than the Macaulay duration. Most strikingly, the duration adjusted for both default and call risks is always shorter than the Macaulay duration. Further diagnosis of the interactive call and default effects shows that controlling for

the effect of default risk, the impact of call risk on duration diminishes as bond rating deteriorates. On the other hand, controlling for the effect of call risk, default risk lengthens duration as bond rating declines and maturity increases. Moreover, our results show that during the period of recession and declining interest rates, the effects of call and default risks on bond duration magnify.

Our study contributes to the literature by providing the direct estimates of the joint and marginal effects of default and call risks on duration. We assess the extent to which default and call risks affect duration by controlling for the effects of other factors.¹ This is accomplished by forming portfolios for riskless straight bonds, riskless callable bonds, defaultable straight bonds, and callable defaultable bonds separately from the Lehman Brothers Fixed Income Database and examining the duration features of these portfolios. In examining the effects of call and default risks on duration, we do not impose any restriction on the term structure of spot yields. As such, we are able to account for the effects of the term structure of interest rates and other bond characteristics on duration simultaneously. This contrasts previous tests (e.g., Sarkar and Hong, 2004) that are constrained by the assumption of a flat yield curve. Finally, unlike prior studies that focus on investment-grade bonds, we examine both investment-grade and speculative bonds with different maturities to assess the effects of call and default risks on duration.

The remainder of this paper is organized as follows. Section II proposes test hypotheses on the effects of different risk factors on bond duration and describes the data sample. Section III presents the empirical models and methodology. Section IV

¹ Previous empirical studies are often restricted by available data. For example, Fons (1990) uses bond indices that include callable and noncallable bonds. Since the call effect is confounded with the default effect, his study may overestimate the effect of default on duration of high-grade bonds. Jacoby and Roberts (2003) bypass Fons's problem but do not measure the effect of the interaction between call and default risks on bond duration.

reports empirical results. Finally, Section V summarizes the findings and concludes the paper.

II. Test Hypotheses and Data

In this section, we first propose test hypotheses on the effects of default risk, call risk and the interaction between these risks on bond duration. Following this, we describe the data and the procedure for hypothesis testing.

1. Test Hypotheses

Denote yields and prices of zero-coupon bonds as y^i and V^i , $i = ct, nc, cc$, where ct stands for callable Treasuries, nc for noncallable corporate bonds, and cc for callable corporate bonds. The pricing function for a zero-coupon bond with the par value of \$1 is

$$(1) \quad V^i = e^{-y^i(T-t)},$$

where $T-t$ is time to maturity. By the elasticity definition, the risk-adjusted duration for a zero-coupon bond is given by $D^i = -(\partial V^i / \partial r)(1/V^i)$ and r is the riskless interest rate. The Macaulay duration is defined as $D^m = -(\partial V^i / \partial y^i)(1/V^i) = T - t$. Substituting (1) into the risk-adjusted duration formula and taking the derivative with respect to the riskless rate, we can rewrite the duration of a zero-coupon bond as

$$(2) \quad D^i = \frac{\partial y^i}{\partial r} (T - t).$$

Since the Macaulay duration of a zero-coupon bond is equal to the time to maturity, $T-t$, equation (2) shows that risk-adjusted duration of a zero-coupon bond is equal to the derivative of the spot yield with respect to the riskless rate multiplied by the Macaulay duration (D^M). If the assumptions underlying the Macaulay duration hold, i.e., a flat yield curve, and no default and call risk, then $\partial y / \partial r = 1$ and (2)

reduces to the Macaulay duration. However, these assumptions are quite stringent. A yield curve could be upward sloping, downward sloping or hump-shaped. Furthermore, most corporate bonds are callable (see Duffee, 1999; Duffie and Singleton, 1999) and thus are subject to both default and call risks. Given the more realistic conditions, equation (2) implies that the call-adjusted, default-adjusted or the default- and call-adjusted duration could be longer or shorter than the Macaulay duration, depending on changes in the spot yield of callable bonds, defaultable bonds, or callable defaultable bonds with respect to changes in the riskless rate. In the following, we propose hypotheses to test the relative effect of call and default risks.

By applying the relation in (2) to different types of bonds, i.e., callable Treasuries and noncallable or callable corporate bonds, we can easily establish the following relations:

$$(3a) \quad D^{ct} = (\partial y^{ct} / \partial r) D^M$$

$$(3b) \quad D^{nc} = (\partial y^{nc} / \partial r) D^M$$

$$(3c) \quad D^{cc} = (\partial y^{cc} / \partial r) D^M$$

These functions summarize the relations between the durations of a pair of bonds of different types. Depending on the magnitude of the derivative term on the right hand side, we can form the following hypotheses.

H1: If $\partial y^{ct} / \partial r < 1$, call risk reduces duration in the absence of default risk, and vice versa.

H2: If $\partial y^{nc} / \partial r < 1$, default risk reduces duration in the absence of call risk, and vice versa.

H3: If $\partial y^{cc} / \partial r < 1$, default and call risks jointly reduce duration, and vice versa.

These hypotheses transform the relation between the risk-adjusted duration and the

Macaulay duration into the relation between the change in the spot yield of a risky bond and the change in the riskless rate. Thus, whether these hypotheses will be accepted or rejected depends on the relation between the change in the spot yield of a risky bond and the change in the riskless rate.

2. Data Description

Since the test hypotheses and empirical models are based on zero-coupon bond yields, we require spot yields to conduct empirical investigation. To estimate the spot yields for different groups of bonds, we first construct indices for noncallable and callable Treasuries with different maturities, and noncallable and callable corporate bonds with different ratings and maturities. Spot yields are then obtained for these bond groups. Treasury and corporate bonds are taken from the Lehman Brothers Fixed Income Database distributed by Warga (1998). This database contains month-end price, accrued interest, and coupons of all Treasury and corporate bonds from January 1973 to December 1996. It also provides descriptive information for quote methods, ratings and callability.

A subset of the database is used in this study. We eliminate all bonds with matrix prices, embedded options except call provision (e.g., puttable bonds and bonds with sinking funds), an odd frequency of coupon payments and floating rates, and bonds not included in the Lehman Brothers Bond Indices. Next, we choose noncallable and callable Treasury bonds with time to maturity less than 15 years to construct noncallable and callable Treasury bond indices. We further decompose these indices into 28 maturity groups from 2 years, 2.5 years, to 15 years, each with 6 months apart. Similarly, we select callable and noncallable industrial corporate bonds with maturities of less than 15 years and divide the sample into 28 maturity groups (2

year, 2.5 years, ..., 15 years) for each rating class, AA, A, BBB and BB. We then use the Nelson-Siegel (1987) procedure to extract monthly spot yields of noncallable and callable Treasury bonds, and noncallable and callable AA to BB bonds with these maturity tranches over the period of January 1986 to December 1996 from coupon bond data (see also Elton et al., 2001).²

Figures 1 and 2 plot mean estimated spot yields of callable and noncallable Treasury and corporate bonds with different maturities. Figure 1 displays the term structure of spot yields of noncallable Treasury and corporate bonds. Spot yields of Treasury bonds are lowest, followed by those of AA bonds and others. As the rating of corporate bonds deteriorates, spot yield gets higher. As shown, spot yields of BB bonds are much higher than those of investment-grade bonds, reflecting a jump in junk bond premium. These empirical term structures reflect the additional premium required by bondholders for bearing higher risk. Yield curves of Treasury and corporate bonds are generally upward sloping. Figure 2 shows the term structure of spot yields of callable corporate bonds. As expected, spot yields of callable bonds are generally higher than those of noncallable counterparts. The higher yield compensates investors for bearing call risk. While the yield curves of investment-grade callable corporate bonds are upward sloping, the yield curve of callable BB bonds is downward sloping.

After obtaining spot yields, we estimate the effects of call and default risks on duration. As summarized by the test hypotheses, these effects depend on the sensitivity of the spot yield of a risky bond to the riskless rate r . The sign and magnitude of the effect of each risk factor on the duration of bonds with different

² Because firms issued few noncallable bonds prior to mid-1980s, we choose the sample data that start from 1986. Also, due to lack of callable Treasury bond data after March 1995, we use the data from January 1986 to March 1995 to estimate the spot yields of callable Treasury bonds. We use the similar sample period and estimation method as in Elton et al. (2001). Our sample selection criteria are in line

ratings and maturities are ultimately an empirical issue.

III. Empirical Methodology

In this section, we propose empirical models to examine the effects of call risk, default risk and their interactions on duration. Hypothesis 1 (H1) states that the relation between the call-adjusted duration and the Macaulay duration depends on the relation between the change in the spot yields of callable Treasury bonds and the change in the riskless interest rate. Since a callable bond is more likely to be called when the interest rate declines, we expect that the duration of a callable Treasury would be affected more significantly during the period of declining interest rate. We hence use the following regression model to examine the effect of call risk on duration:

$$(4) \quad y_m^{ct} = \mathbf{b}_{0,m} + \mathbf{b}_{1,m} \times r + \mathbf{b}_{2,m} \times r \times \text{DIR} + \mathbf{b}_{3,m} \times \text{DIR} + \mathbf{e}_m,$$

where y_m^{ct} is the monthly continuously compounded spot yields of callable Treasury bonds with maturity m ,³ and m = short term (2-5 years), medium term (5-10 years), or long term (10-15 years), r is the monthly continuously compounded spot yields of noncallable Treasury bonds with maturity m , DIR is a dummy variable which is equal to 1 if an observation falls in the period with declining interest rates and 0, otherwise, and \mathbf{e}_m is an error term. During the period of our data sample (1986-1996), we find that interest rates continuously decline over the period of 1990 to 1993. Hence, in equation (4), DIR = 1 if an observation falls in the period from 1990 to 1993 and 0, otherwise. Since the theoretical relation between the duration of a callable Treasury bond (D^c) and its Macaulay counterpart (D^M) is given by $D^c/D^M = \partial y_m^{ct}/\partial r$, the slope coefficient of r in equation (4) is given by $\partial y_m^{ct}/\partial r = \mathbf{b}_{1,m}$ if the interest rate does not

with theirs and as a result, our estimates are comparable to theirs.

decline, and $\partial y_m^{ct}/\partial r = \mathbf{b}_{1,m} + \mathbf{b}_{2,m}$ if the interest rate declines. The slope coefficient of r directly thus measures the duration ratio of D^c/D^M . As a result, testing the effect of call risk on duration in H1 is equivalent to testing whether $\mathbf{b}_{1,m}$ is different from one and whether $\mathbf{b}_{2,m}$ is different from 0 if interest rate is declining. If $\mathbf{b}_{1,m} < 1$, it implies that call risk shortens duration, and vice versa. If $\mathbf{b}_{2,m} < 0$, call risk further shortens duration in the period of declining interest rates, and vice versa.

Similarly, we can employ the regression model to examine the effect of default risk on duration in H2. Based on the option model, a defaultable bond can be treated as a risk-free bond embedded with a put option. That is, shareholders have the right to “put” a firm to bondholders if the firm value drops below the debt value. Typically, the put option is more likely to be exercised during the economic downturn. Therefore, we expect that the duration of defaultable bonds to be more significantly affected during the recession. To test these implications, we propose the following regression model:

$$(5) \quad y_m^{nc} = \mathbf{b}_{0,m} + \mathbf{b}_{1,m} \times r + \mathbf{b}_{2,m} \times r \times \text{Rec} + \mathbf{b}_{3,m} \times \text{Rec} + \mathbf{e}_m,$$

where y_m^{nc} is the monthly continuously compounded spot yields of noncallable corporate bond index ($nc = AA, A, BBB, BB$) with maturity m , $\text{Rec} = 1$ if economy is in recession and 0, otherwise while other terms are defined as before. Note that U.S. economy was in recession during the period of 1990 to 1993. Thus, in equation (5), $\text{Rec} = 1$ if an observation falls in the period from 1990 to 1993 and 0 otherwise. A test of the effect of default risk on duration is equivalent to a test of whether $\mathbf{b}_{1,m}$ is different from one in a normal growth economy, and whether $\mathbf{b}_{2,m}$ is different from 0 if economy is in recession. If $\mathbf{b}_{1,m} < 1$, it implies that default risk shortens duration. If

³ The continuously compounded spot yield is calculated as log of one plus the discrete spot yield.

$b_{2,m} < 0$, default risk further shortens duration in the period of recession.

Hypothesis 3 (H3) states that the relation between the duration of a callable defaultable bond and its Macaulay counterpart depends on the relation between the spot yield of the callable defaultable bond and riskless interest rate. We employ the following regression model to examine the effect of the interaction between default and call risks on duration:

$$(6) \quad y_m^{cc} = b_{0,m} + b_{1,m} \times r + b_{2,m} \times r \times \text{DIR_Rec} + b_{3,m} \times \text{DIR_Rec} + e_m,$$

where y_m^{cc} is the monthly continuously compounded spot yields of callable corporate bond index with different ratings ($cc = \text{callable AA, A, BBB, BB}$) and maturity m , $\text{DIR_Rec} = 1$ if an observation falls in the period between 1990 and 1993 and 0 otherwise, and other terms are defined as before. A test of the effect of the interaction between default and call risks on duration is equivalent to a test of whether $b_{1,m}$ is different from one and whether $b_{2,m}$ is different from 0 if the economy is in recession with declining interest rates. If $b_{1,m} < 1$, it implies that default and call risks jointly shortens duration. If $b_{2,m} < 0$, default and call risks further shortens duration in the period of recession with declining interest rate, and vice versa.

The regression model is estimated by time-series yield data. A typical problem encountered in empirical estimation is that the residuals of yield regressions exhibit strong serial correlation. We employ the Yule-Walker method to account for the effect of serial correlation when fitting the regression model.

IV. Empirical Results

Tables 1 to 3 report the results of regression models (4) to (6), respectively. Each table shows estimates of regression parameters, t -values, adjusted R^2 , and the Durbin-Watson statistic adjusted for serial correlation.

Table 1 presents the results for the effect of call risk on duration. b_1 is a direct estimate of the ratio of duration of callable Treasuries to its Macaulay counterpart (D^c/D^M). This coefficient is significantly less than one at the one percent level for all maturity groups. Moreover, b_2 for the medium-term bonds is significantly less than zero. Results suggest that durations of callable Treasury bonds are shorter than their Macaulay counterparts, and that during the period of declining interest rates, call risk further shortens the duration of the medium-term callable bonds. Our finding is consistent with the theoretical prediction of Acharya and Carpenter (2002) that a call provision reduces duration in the absence of default risk. Their option-based model treats a callable bond as a combination of a long noncallable riskless host bond and a short call option on that bond with call price as the exercise price. If, say, noncallable bond price rises, price of callable bonds should not rise as much because the value of the embedded short call option also rises. Since bond price and spot yield have an inverse relationship, we expect that if the spot yield of noncallable Treasury bonds rises, the spot yield of callable Treasury bonds should not rise as much. The results in Table 1 support this contention and suggest that call risk reduces duration. Our results are also consistent with Dunetz and Mahoney's (1988) finding that the duration of callable bonds is lower than the duration of those bonds measured under the assumption that they are held to maturity. Furthermore, our results imply that medium-term Treasury bonds (5 to 10 years) are more likely to be called when interest rates decline as reflected by the shorter expected duration of these bonds during the period of falling interest rates.⁴

[Insert Table 1 about here]

Table 2 reports the results of the effect of default risk on the duration for

⁴ This finding may reflect that most Treasury bonds have maturity less than 10 years in our data

bonds with different maturities and credit ratings. The empirical results for the default effect are mixed. b_j 's (or duration ratios) for medium- and long-term BBB bonds, and short- and medium-term BB bonds are significantly lower than one, suggesting that default risk shortens the duration of these bonds. By contrast, b_j 's for short- and medium-term AA bonds and short-term A bonds are significantly greater than one, suggesting that the default risk-adjusted duration is longer than the Macaulay counterpart. These findings are consistent with the prediction of the models by Nawalkha (1996), and Xie, Liu and Wu (2005). In particular, Xie et al. (2005) show that under the assumption of the dependence between interest rate and firm value or default intensity, the default risk-adjusted duration can be longer than, equal to, or shorter than the Macaulay duration. By contrast, our results only partially support Acharya and Carpenter's (2002) contention that default risk alone reduces duration. Our empirical results show that default risk shortens duration only for lower-grade bonds. One possible reason is that the default risk for lower-grade bonds is higher and so its negative effect on duration is more significant. Conversely, the default risk for shorter-term AA and A bonds is negligible and so the negative effect of default risk on the duration of these bonds is considerably weakened. Finally, results show that during the period of recession, default risk shortens the duration of short-term A and BBB bonds, but it lengthens the duration of medium- and long-term AA bonds and long-term A bonds.

[Insert Table 2 about here]

Table 3 shows the results of the combined effect of default and call risks on duration. b_j 's are all significantly less than one at the one percent level for all rated bonds with different maturities. Results indicate that durations of callable corporate

sample.

bonds are significantly shorter than their Macaulay counterparts, suggesting that call risk interacts with default risk to reduce duration. Additionally, during the period of recession with declining interest rates, the interaction between call and default risks further shortens the duration of callable corporate bonds. Interestingly, we find that the estimated b_j 's in Table 3 are smaller than those in Table 2. The difference is larger for high-grade bonds, such as AA and A bonds, than for low-grade bonds. As shown in Table 2, for long-term AA and medium- and long-term A bonds, default risk alone has no significant impact on their durations. However, when call risk interacts with default risk, their risk-adjusted durations are significantly shorter than their Macaulay counterparts. The results suggest that call risk is a more influential adjustment factor for high-grade bonds than default risk. This evidence is consistent with the finding of Jacoby and Roberts (2003) that the call effect generally dominates the default risk effect on bond durations. It is also consistent with the finding of Sarkar and Hong (2004) that call risk adjustment for duration is more important for high-grade bonds. On the other hand, our results question Fons' (1990) argument that default risk always reduces the durations of corporate bond indices. As noted earlier, corporate bond indices used by Fons (1990) are composed of both callable and noncallable bonds. Our results suggest that the shorter empirical duration of corporate bond indices documented by Fons (1990) are predominantly attributed to the call risk effect rather than the default risk effect.

[Insert Table 3 about here]

Although the results in Table 3 and Table 2 suggest that call risk is a more important adjustment factor for the duration of callable corporate bonds than default risk, the magnitude of the incremental effect of call risk on the duration of callable corporate bonds is still unclear. To capture this incremental (or marginal) call risk

effect, we estimate the following regression model:

$$(7) \quad \begin{aligned} y_m^{cc} - y_m^{nc} &= (\mathbf{b}_{0,m}^{cc} - \mathbf{b}_{0,m}^{nc}) + (\mathbf{b}_{1,m}^{cc} - \mathbf{b}_{1,m}^{nc}) \times r + (\mathbf{b}_{2,m}^{cc} - \mathbf{b}_{2,m}^{nc}) \times r \times \text{DIR_Rec} + \mathbf{e}_m^* \\ &= \mathbf{b}_{0,m}^* + \mathbf{b}_{1,m}^* \times r + \mathbf{b}_{2,m}^* \times r \times \text{DIR_Rec} + \mathbf{e}_m^*. \end{aligned}$$

where $\mathbf{b}_{1,m}^*$ measures the incremental call effect on duration of a defaultable bond, and $\mathbf{b}_{2,m}^*$ captures the incremental call effect on duration during the period with declining interest rates. Negative $\mathbf{b}_{1,m}^*$ and $\mathbf{b}_{2,m}^*$ values imply that call risk reduces bond duration after controlling the effect of default risk.

Table 4 shows that the estimates of $\mathbf{b}_{1,m}^*$ for investment-grade bonds with different maturities are all less than zero and significant at the one percent level, suggesting that controlling for the default risk, call risk further reduces the duration of investment-grade callable bonds with different maturities. For example, $\mathbf{b}_{1,m}^*$ of short-term AA bond is -0.2792, meaning that call risk further reduces the duration of short-term AA bond by 0.2792. Conversely, the estimates of $\mathbf{b}_{1,m}^*$ for speculative-grade bonds with different maturities are not significantly different from zero, implying that call risk has no significant incremental effect on the duration of speculative bonds. On the other hand, we do not find that call risk significantly lengthens the duration of speculative bonds either. Our results are consistent with Sarkar and Hong's (2004) argument that the call feature shortens duration for high-grade bonds but it has no significant effect on duration of speculative bonds. More importantly, our results strongly support Acharya and Carpenter's (2002) contention that when considering both default and call risks, the effect of the call provision on bond duration diminishes as the probability of default increases. Furthermore, we find that call risk further reduces the duration of medium-term and long-term investment-grade bonds during the period of recession with declining interest rates but has no noticeable effect on the

duration of speculative-grade bonds. This finding is also consistent with the prediction of Acharya and Carpenter (2002).

[Insert Table 4 about here]

To further investigate the impact of the interaction between default and call risks on duration, we estimate the incremental effect of default risk on duration in presence of call risk. Specifically, we use the callable AA bond as the reference bond and compare the duration of callable A, BBB, and BB bonds to the duration of the reference bond. That is, $D^i / D^{cAA} = \partial y^i / \partial y^{cAA}$, $i =$ callable A, BBB, and BB bonds. We then use the following regression model to test whether $\partial y^i / \partial y^{cAA}$ is greater than, equal to, or less than one:

$$(8) \quad y_m^i = \mathbf{b}_{0,m} + \mathbf{b}_{1,m} \times y_m^{cAA} + \mathbf{b}_{2,m} \times y_m^{cAA} \times \text{Rec} + \mathbf{b}_{3,m} \times \text{Rec} + \mathbf{e}_m,$$

where y_m^i stands for the spot yields of callable A, BBB, and BB bonds with maturity m , y_m^{cAA} for the spot yields of callable AA bonds with maturity m , and other variables are defined as before. $\mathbf{b}_{1,m}$ measures the ratio of the duration of callable bond i to that of AA bond, and $\mathbf{b}_{2,m}$ captures the change in this ratio during the period of recession.

Table 5 shows that the estimates of \mathbf{b}_1 are less than one, except for medium-term BB bonds and long-term BBB and BB bonds. The results indicate that the durations of short-term callable A, BBB, and BB bonds, and long-term A bonds are shorter than the duration of AA bonds. While default risk generally decreases the duration of risky bonds in presence of call risk, it can increase the duration of longer-term lower-grade bonds. These findings are consistent with the predictions of Acharya and Carpenter (2002). Their model predicts that when both call and default risks are present, default risk may increase the duration of a callable bond because the exercise of one option rules out the exercise of the other option. Our results suggest that this

trade-off effect is stronger for long-term lower-grade bonds. For bonds with higher default risk, the issuer is more likely to exercise the default option than the call option. The decrease in the probability of exercising the call option lengthens bond duration whereas the increase in the probability of exercising the default option shortens duration. Because the incremental (or marginal) effect of call risk on duration is greater than that of default risk, bond duration ends up higher when the probability of exercising the call option becomes lower as default risk increases for lower grade bonds. Finally, the results in Table 5 show that $b_{2,m}$'s are all negative. This suggests that during the period of economic downturn, higher default risk further reduces the durations of callable bonds.

[Insert Table 5 about here]

V. Conclusion

In this paper we test the implications of Acharya and Carpenter's model for the joint and marginal effects of default and call risks on bond duration. To accomplish this task, we formulate empirical models to examine the effects of call risk, default risk, and their interactions on bond duration using the Lehman Brothers Fixed Income Database. We find that the empirical duration of callable Treasury bonds is lower than its Macaulay counterpart, indicating that call risk alone reduces duration. During the period of declining interest rates, call risk further shortens the duration of medium-term callable Treasury bonds. With respect to default risk, we find that default risk may increase or decrease bond duration.

Our paper adds value to the literature by documenting a significant interactive effect of call and default risks on bond duration. Although default risk alone can shorten or lengthen duration of risky bonds, when call risk interacts with default risk, the durations of defaultable callable bonds are always lower than their

Macaulay counterparts. This interactive effect is stronger during the recession period with declining interest rates. Our results suggest that call risk is a more important factor for duration adjustment.

Another contribution of our paper is the documentation of the incremental effects of call and default risk on duration. Controlling for the effect of default risk, we find that call risk shortens the duration of investment-grade bonds with different maturities, whereas it has no significant impact on the duration of speculative bonds. The results indicate that as default risk increases, the effect of call risk on duration diminishes. Controlling for the effect of call risk, default risk generally decreases the duration of higher-grade bonds but can increase the duration of longer-term lower-grade bonds. One reason that the incremental effect of default risk is positive for lower-grade bonds is because higher default risk leads to a lower probability of exercising the call option. As suggested by Acharya and Carpenter (2002), default risk can increase bond duration under this circumstance. Their model shows that when both default and call risks are present, the effect of call option on bond duration diminishes as default risk increases. Because the marginal effect of call option on duration is stronger than that of default option, the former effects outweighs the later, resulting in a net increase in duration when the probability of exercising the call option decreases for higher-risk bonds.

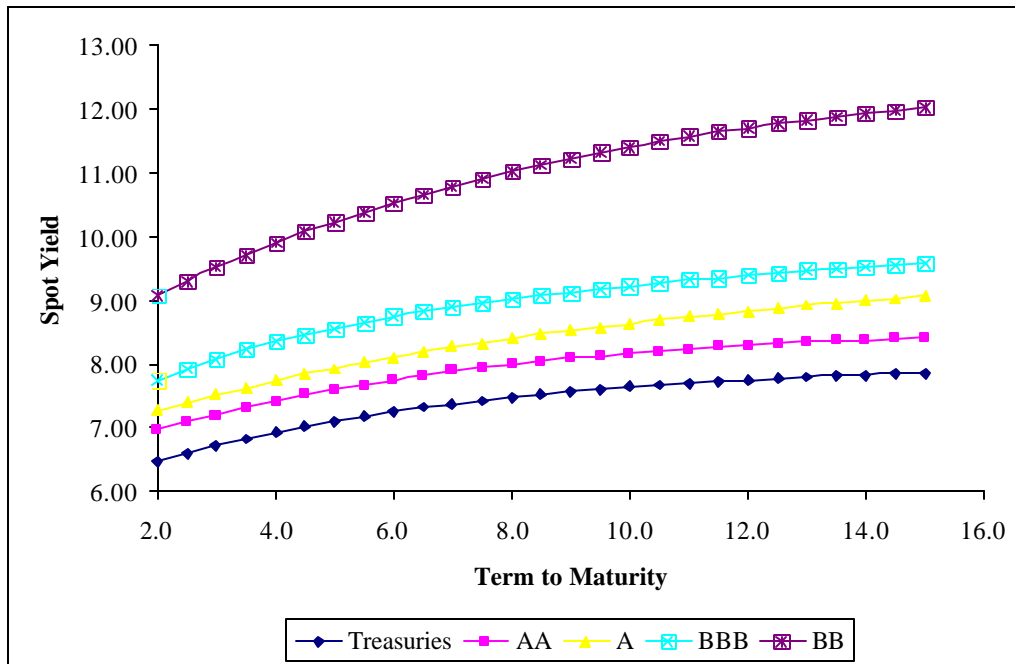
Our findings suggest that to formulate an effective hedging strategy, portfolio managers must adjust the Macaulay duration for the interactive effects between call and default risks. Our study provides useful information for portfolio managers to gauge the sensitivity of duration to either call or default risk at the margin for corporate bonds of different ratings and maturities. The simple empirical models we propose to estimate the incremental effects of call and default risks on duration can be

used easily by portfolio managers to assess the marginal impact of each risk factor and to adjust the duration for their bond portfolio accordingly to achieve an efficient hedge. Furthermore, portfolio managers should pay special attention to the period of economic downturns with declining interest rates because the effects of default and call risks on duration tend to magnify.

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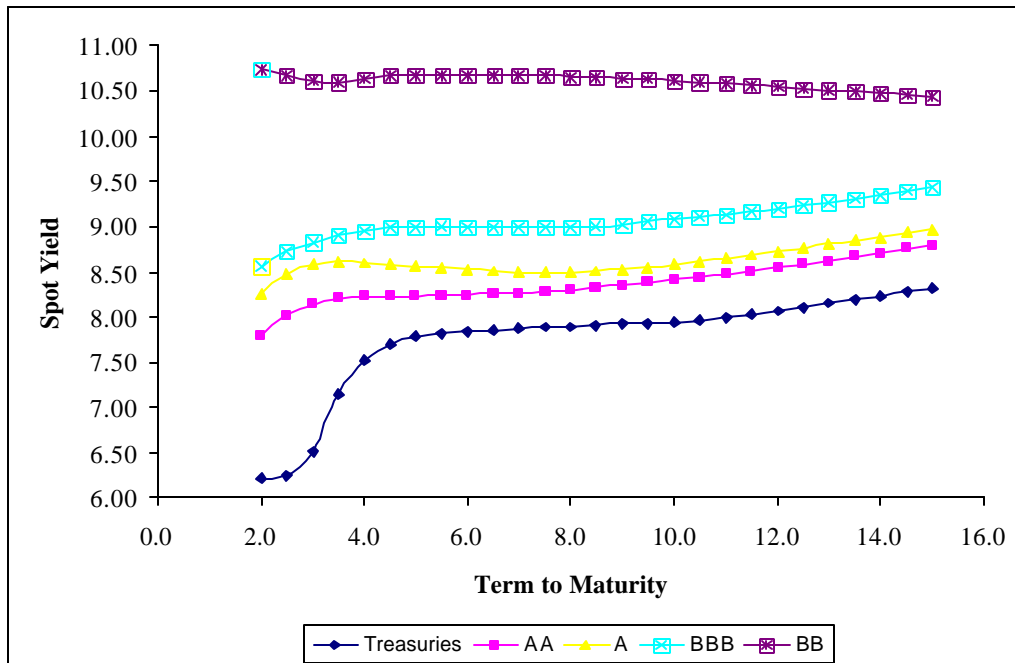
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Figure 1. Spot Yields of Noncallable Treasury and Corporate Bonds



This figure displays the term structure of estimated spot yields of noncallable Treasury and corporate bonds. The spot yields for each maturity across different ratings are averaged over the period of January 1986 to December 1996.

Figure 2. Spot Yields of Callable Corporate Bonds



This figure displays the term structure of estimated spot yields of callable corporate bonds. The spot yields for each maturity across different ratings are averaged over the period of January 1986 to December 1996 whereas the callable Treasury spot yields are averaged over the period of January 1986 to March 1995.

Table 1
Effect of Call Risk on Duration

This table reports the estimates of the relation between the duration of callable zero-coupon bonds and its Macaulay counterpart from the equation $y_m^{ct} = \beta_{0,m} + \beta_{1,m} \times r + \beta_{2,m} \times r \times \text{DIR} + \beta_{3,m} \times \text{DIR} + e_m$ for the period: 01:1986 - 03:1995, where y_m^{ct} is the monthly continuously compounded spot yields of the callable Treasury bonds with maturity m , $m =$ short-term (2-5 years), medium-term (5-10 years), and long-term (10-15 years). r is the monthly continuously compounded spot yields of noncallable Treasury bond index with maturity m . $\text{DIR} = 1$ if interest rates decline (1990-1993) and 0 otherwise, and e_m is an error term. Also reported are the adjusted R^2 and the Durbin-Watson statistic (DW) adjusted for the serial correlation. The t -statistic for β_1 is used to test the null hypothesis that the coefficient is different from the unity and the t -statistics for other coefficients are used to test the null hypothesis that the coefficients are different from zero. ** indicates significance at the 1% level.

Maturity	Model	β_0	β_1	β_2	β_3	DW	Adj. R^2
Short	Coefficient	0.0163	0.7629	-0.0413	0.0026	2.00	0.93
	t -Statistic	6.04**	-6.99**	-0.98	0.95		
Medium	Coefficient	0.0199	0.7589	-0.1088	0.0078	2.04	0.94
	t -Statistic	19.56**	-17.99**	-5.68**	5.80**		
Long	Coefficient	0.0145	0.8259	0.0123	-0.0008	1.98	0.98
	t -Statistic	20.89**	-19.78**	0.82	-0.69		

Table 2
Effect of Default Risk on Duration

This table reports the estimates of the relation between the duration of defaultable zero-coupon bonds and its Macaulay counterpart from the equation $y_m^{nc} = \mathbf{b}_{0,m} + \mathbf{b}_{1,m} \times r + \mathbf{b}_{2,m} \times r \times \text{Rec} + \mathbf{b}_{3,m} \times \text{Rec} + \mathbf{e}_m$ for the period: 01:1986 - 12:1996, where $nc = AA, A, BBB,$ and BB ; $m =$ short-term (2-5 years), medium-term (5-10 years), and long-term (10-15 years). y_m^{nc} is the monthly continuously compounded spot yields of the noncallable corporate bond index with maturity m . r is the monthly continuously compounded spot yields of noncallable Treasury bonds with maturity m . $\text{Rec} = 1$ if economy is in recession (1990-1993) and 0 otherwise, and \mathbf{e}_m is an error term. Also reported are the adjusted R^2 and the Durbin-Watson statistic (DW) adjusted for the serial correlation. The t -statistic for \mathbf{b}_1 is used to test the null hypothesis that the coefficient is different from the unity and the t -statistics for other coefficients are used to test the null hypothesis that the coefficients are different from zero. ** and * indicate significance at the 1% and 5% levels, respectively.

Maturity	Rating	Model	β_0	β_1	β_2	β_3	DW	Adj. R^2
Short	AA	Coefficient	-0.0003	1.0681	-0.0133	0.0019	2.00	0.92
		t -Statistic	-0.22	3.98**	-0.54	1.17		
Short	A	Coefficient	0.0027	1.0576	-0.0787	0.0078	2.00	0.97
		t -Statistic	3.44**	5.14**	-4.89**	7.34**		
Short	BBB	Coefficient	0.0101	1.0119	-0.0786	0.0101	1.95	0.92
		t -Statistic	5.33**	0.43	-1.99*	3.94**		
Short	BB	Coefficient	0.0317	0.8179	0.0131	0.0177	2.07	0.64
		t -Statistic	5.36**	-2.31**	0.1066	2.20*		
Medium	AA	Coefficient	-0.0006	1.0701	0.1406	-0.0090	1.99	0.72
		t -Statistic	-0.31	2.68**	3.34**	-2.98**		
Medium	A	Coefficient	0.0060	1.0267	0.0147	0.0002	2.00	0.92
		t -Statistic	4.45**	1.45	0.49	0.11		
Medium	BBB	Coefficient	0.0171	0.9440	-0.0238	0.0044	1.93	0.90
		t -Statistic	8.80**	-2.12*	-0.54	1.41		
Medium	BB	Coefficient	0.0485	0.6958	0.2032	-0.0010	2.02	0.60
		t -Statistic	7.11**	-3.27**	1.32	-0.09		
Long	AA	Coefficient	0.0077	0.9520	0.3075	-0.0204	1.99	0.36
		t -Statistic	2.02*	-0.95	3.35**	-2.97**		
Long	A	Coefficient	0.0111	0.9922	0.2354	-0.0185	2.03	0.69
		t -Statistic	3.33**	-0.18	2.85*	-3.02**		
Long	BBB	Coefficient	0.0267	0.8488	0.1507	-0.0118	1.82	0.82
		t -Statistic	7.54**	-3.24**	1.68	-1.79		
Long	BB	Coefficient	0.0321	1.0139	-0.1745	0.0208	2.01	0.57
		t -Statistic	3.48**	0.11	-0.75	1.21		

Table 3
Effect of Default and Call Risks on Duration

This table reports the estimates of the relation between the duration of callable defaultable zero-coupon bonds and its Macaulay counterpart from the equation $y_m^{cc} = \mathbf{b}_{0,m} + \mathbf{b}_{1,m} \times r + \mathbf{b}_{2,m} \times r \times \text{DIR_Rec} + \mathbf{b}_{3,m} \times \text{DIR_Rec} + \mathbf{e}_m$ for the period: 01:1986 - 12:1996, where $cc = \text{callable AA, A, BBB, and BB}$; $m = \text{short-term (2-5 years), medium-term (5-10 years), and long-term (10-15 years)}$. y_m^{cc} is the monthly continuously compounded spot yields of the callable corporate bond index with maturity m . r is the monthly continuously compounded spot yields of noncallable Treasury bonds. $\text{DIR_Rec} = 1$ if year is 1990 to 1993 and 0 otherwise, and \mathbf{e}_m is an error term. Also reported are the adjusted R^2 and the Durbin-Watson statistic (DW) adjusted for the serial correlation. The t -statistic for \mathbf{b}_1 is used to test the null hypothesis that the coefficient is different from the unity and the t -statistics for other coefficients are used to test the null hypothesis that the coefficients are different from zero. ** and * indicate significance at the 1% and 5% levels, respectively.

Maturity	Rating	Model	β_0	β_1	β_2	β_3	DW	Adj. R^2
Short	AA	Coefficient	0.0301	0.7354	-0.0100	-0.0005	1.90	0.98
		t -Statistic	21.58**	-16.04**	-0.43	-0.35		
Short	A	Coefficient	0.0391	0.6468	-0.0664	0.0043	1.69	0.98
		t -Statistic	36.13**	-26.96**	-3.62**	3.59**		
Short	BBB	Coefficient	0.0400	0.6806	-0.0711	0.0042	1.99	0.97
		t -Statistic	31.67**	-19.72**	-3.12*	2.85**		
Short	BB	Coefficient	0.0646	0.5431	-0.1078	0.0086	2.01	0.86
		t -Statistic	19.63**	-9.80**	-1.63	2.01*		
Medium	AA	Coefficient	0.0309	0.6854	-0.0259	0.0013	1.89	0.99
		t -Statistic	56.89**	-44.10**	-2.15*	1.53		
Medium	A	Coefficient	0.0348	0.6584	-0.0369	0.0024	1.75	0.99
		t -Statistic	62.55**	-49.37**	-3.14**	2.97**		
Medium	BBB	Coefficient	0.0380	0.6762	-0.0618	0.0040	1.91	0.98
		t -Statistic	52.22**	-35.66**	-4.02**	3.78**		
Medium	BB	Coefficient	0.0479	0.7393	-0.0684	0.0059	1.89	0.92
		t -Statistic	33.57**	-13.44**	-2.10*	2.58**		
Long	AA	Coefficient	0.0332	0.6608	0.0380	-0.0033	2.02	0.97
		t -Statistic	54.47**	-42.36**	2.43*	-2.91**		
Long	A	Coefficient	0.0360	0.6415	0.0251	-0.0018	1.91	0.98
		t -Statistic	67.11**	-50.97**	1.82	-1.77		
Long	BBB	Coefficient	0.0338	0.7324	-0.0742	0.0049	2.07	0.97
		t -Statistic	47.13**	-28.57**	-4.00**	3.63**		
Long	BB	Coefficient	0.0422	0.7801	-0.1174	0.0073	2.03	0.92
		t -Statistic	35.82**	-14.19**	-3.91**	3.30**		

Table 4
Incremental Call Effect on Duration in the Presence of Default Risk

This table reports the incremental call effect on the duration of callable defaultable bonds estimated from the equation

$$y_m^{cc} - y_m^{nc} = (\mathbf{b}_{0,m}^{cc} - \mathbf{b}_{0,m}^{nc}) + (\mathbf{b}_{1,m}^{cc} - \mathbf{b}_{1,m}^{nc}) \times r + (\mathbf{b}_{2,m}^{cc} - \mathbf{b}_{2,m}^{nc}) \times r \times \text{DIR_Rec} + \mathbf{e}_m^*$$

$$= \mathbf{b}_{0,m}^* + \mathbf{b}_{1,m}^* \times r + \mathbf{b}_{2,m}^* \times r \times \text{DIR_Rec} + \mathbf{e}_m^*$$

over the period : 01:1986 - 12:1996, where *cc* = *callable AA, A, BBB, and BB*; *nc* = *AA, A, BBB, and BB*; *m* = short-term (2-5 years), medium-term (5-10 years), and long-term (10-15 years). y_m^{cc} is the monthly continuously compounded spot yields of the callable corporate bond index with maturity *m*. y_m^{nc} is the monthly continuously compounded spot yields of the noncallable corporate bond index with maturity *m*. *r* is the monthly continuously compounded spot yields of noncallable Treasury bonds. DIR_Rec = 1 if year is 1990 to 1993 and 0 otherwise, and \mathbf{e}_i^* is an error term. Also reported are the adjusted R^2 and the Durbin-Watson statistic (*DW*) adjusted for the serial correlation. The *t*-statistics for coefficients are used to test the null hypothesis that the coefficients are different from zero. ** and * indicate significance at the 1% and 5% levels, respectively.

Maturity	Rating	Model	β_0^*	β_1^*	β_2^*	β_3^*	DW	Adj. R ²
Short	AA	Coefficient	0.0272	-0.2792	-0.0591	0.0003	2.09	0.58
		<i>t</i> -Statistic	11.00**	-7.85**	-1.16	0.09		
Short	A	Coefficient	0.0257	-0.2574	-0.0207	0.0005	2.00	0.83
		<i>t</i> -Statistic	15.91**	-11.47**	-0.65	0.23		
Short	BBB	Coefficient	0.0265	-0.2900	-0.0152	-0.0028	2.00	0.66
		<i>t</i> -Statistic	12.42**	-9.45**	-0.35	-0.98		
Short	BB	Coefficient	0.0156	-0.0586	-0.1209	-0.0005	1.96	0.50
		<i>t</i> -Statistic	2.24*	-0.58	-0.83	-0.05		
Medium	AA	Coefficient	0.0245	-0.2886	-0.2537	0.0168	1.99	0.25
		<i>t</i> -Statistic	11.84**	-10.23**	-5.59*	5.18**		
Medium	A	Coefficient	0.0244	-0.3099	-0.0865	0.0053	2.05	0.66
		<i>t</i> -Statistic	17.83**	-16.63**	-2.81**	2.45*		
Medium	BBB	Coefficient	0.0187	-0.2364	-0.0480	0.0005	1.93	0.66
		<i>t</i> -Statistic	10.33**	-9.61**	-1.17	0.17		
Medium	BB	Coefficient	-0.0027	0.0730	-0.2229	0.0036	2.01	0.51
		<i>t</i> -Statistic	-0.41	0.82	-1.52	0.35		
Long	AA	Coefficient	0.0194	-0.2099	-0.2828	0.0179	1.99	0.07
		<i>t</i> -Statistic	4.95**	-4.06**	-3.00**	2.54*		
Long	A	Coefficient	0.0212	-0.3001	-0.2068	0.0163	2.05	0.32
		<i>t</i> -Statistic	6.96**	-7.48**	-2.75**	2.93**		
Long	BBB	Coefficient	0.0087	-0.1380	-0.2052	0.0154	1.85	0.62
		<i>t</i> -Statistic	2.59**	-3.11**	-2.42*	2.45*		
Long	BB	Coefficient	0.0099	-0.2295	0.0500	-0.0132	1.99	0.50
		<i>t</i> -Statistic	1.10	-1.94	0.22	-0.79		

Table 5

Incremental Default Effect on Duration in the Presence of Call Risk

This table reports the incremental default effect on the duration of callable defaultable bonds estimated from the following equation:

$$y_m^i = \mathbf{b}_{0,m} + \mathbf{b}_{1,m} \times y_m^{cAA} + \mathbf{b}_{2,m} \times y_m^{cAA} \times \text{Rec} + \mathbf{b}_{3,m} \times \text{Rec} + \mathbf{e}_m$$

over the period: 01:1986 - 12:1996, where y_m^i stands for the monthly continuously spot yields of callable A, BBB, and BB bonds with maturity m , y_m^{cAA} for the monthly continuously spot yields of callable AA bonds with maturity m , m = short-term (2-5 years), medium-term (5-10 years), and long-term (10-15 years), Rec = 1 if economy is in recession (1990-1993) and 0 otherwise, and \mathbf{e}_m is an error term. Also reported are the adjusted R^2 and the Durbin-Watson statistic (DW) adjusted for the serial correlation. The t -statistic for \mathbf{b}_l is used to test the null hypothesis that the coefficient is different from the unity and the t -statistics for other coefficients are used to test the null hypothesis that the coefficients are different from zero. ** and * indicate significance at the 1% and 5% levels, respectively.

Maturity	Rating	Model	β_0	β_1	β_2	β_3	DW	Adj. R^2
Short	A	Coefficient	0.0158	0.8375	-0.0864	0.0080	1.96	0.98
		t -Statistic	16.57**	-13.89**	-4.37**	5.37**		
Short	BBB	Coefficient	0.0207	0.8129	-0.0775	0.0071	2.06	0.98
		t -Statistic	15.83**	-12.15**	-3.12**	3.81**		
Short	BB	Coefficient	0.0425	0.7316	-0.0664	0.0089	2.06	0.87
		t -Statistic	10.91**	-5.63**	-0.83	1.48		
Medium	A	Coefficient	0.0055	0.9547	-0.0287	0.0026	1.98	0.99
		t -Statistic	12.31**	-8.23**	-2.93**	3.41**		
Medium	BBB	Coefficient	0.0074	0.9869	-0.0541	0.0045	1.95	0.99
		t -Statistic	10.40**	-1.50	-3.52**	3.73**		
Medium	BB	Coefficient	0.0166	1.0523	-0.0541	0.0060	1.91	0.93
		t -Statistic	8.68**	2.21*	-1.29	1.82		
Long	A	Coefficient	0.0070	0.9321	-0.0023	0.0007	1.91	0.99
		t -Statistic	10.87**	-8.77**	-0.17	0.64		
Long	BBB	Coefficient	0.0027	1.0395	-0.1174	0.0095	1.99	0.97
		t -Statistic	2.57**	3.13**	-5.38**	5.42**		
Long	BB	Coefficient	0.0150	1.0354	-0.2017	0.0156	2.03	0.91
		t -Statistic	7.58**	1.49	-4.88**	4.65**		