# Fixed Income Securities and Markets

Chapter 4

Generalizations and Curve Fitting

## Overview

- Cash flow not on even six-month intervals:
- Accrued interest
- Compounding conventions other than semiannual
- Curve fitting techniques to estimate discount factors for any time horizon

## Accrued Interest

- On Feb. 15, 2001, consider the 5.5s of Jan. 31, 2003 (Jan. 31 and July 31 coupon payments)
- If Investor B buys $10,000 face value of this bond from investor S,

## Accrued Interest

- 166 days between Feb. 15, 2001 and July 31, 2001
- 181 days between Jan. 31, 2001 and July 31, 2001
- Thus B should receive only (166/181)X$275 = $ 252.21 of the coupon payment.
- S gets the rest: $22.79

## Accrued Interest

- S should receive the rest of the coupon payment, $22.79
- Investor B pays $22.79 of accrued interest to investor S on Feb. 15, 2001, the settlement date.
- Having paid S $22.79, investor B may keep the entire $275 coupon payment on July 31, 2001.

## Flat price and full price

- Quoted or Flat price: 101-4.625
- Accrued Interest: $22.79 per $10,000 face value or .2279%
- Full price: 101+4.625/32+.2279=101.3724
- Invoice price on $10,000 face amount: $10,137.24 (actual amount paid)

- Flat Price + Accrued Interest = PV (future cash flows)

- With an accrued interest convention, if yield does not change then the quoted price of a bond does not fall as a result of a coupon payment.
- Full price does fall.

**PB and PA: quoted prices of a bond right before and right after a coupon payment of c/2.**

- PB+c/2=c/2+PV(cash flows after the next coupon)
- PB=PV(cash flows after the next coupon)
- PA+0=PV(cash flows after the next coupon)
- PA=PB

## Continuous Compounding

**TABLE 6.3**

Compounding Frequency and Effective Annual Rates

| Compounding Period | Number of Times Compounded | Effective Annual Rate |
|---|---|---|
| Year | 1 | 10.00000% |
| Quarter | 4 | 10.38129 |
| Month | 12 | 10.47131 |
| Week | 52 | 10.50648 |
| Day | 365 | 10.51558 |
| Hour | 8,760 | 10.51703 |
| Minute | 525,600 | 10.51709 |

Continuous Compounding: EAR = $e^q - 1$ $= 2.71828^{.10} - 1 = 10.51709\%$

How much will $1 become after t years ???

## Simple Interest

- No interest on interest
- For example, in the money market there is an actual/360 convention.
- Lending $1 for d days at a rate of r will earn the lender an interest payment of (rd)/360 dollars at the end of the d days.

## Linear yield interpolation

- A common but unsatisfactory technique.
- A list of bond spanning the maturity range of interest.
- Bonds best suited for this purpose are those that sell near par and those liquid enough to generate accurate price quotations.
- Construct a par yield curve (yields on par bonds) by connecting the yield of these bonds with straight line.

**TABLE 4.2** Bonds Selected for Linear Yield Interpolation

| Coupon | Maturity | Yield | Price |
|---|---|---|---|
| 5.500% | 07/31/01 | 5.001% | 100.219 |
| 5.875% | 11/30/01 | 4.964% | 100.688 |
| 5.625% | 11/30/02 | 4.885% | 101.244 |
| 5.250% | 08/15/03 | 4.887% | 100.844 |
| 5.875% | 11/15/04 | 4.988% | 102.988 |
| 6.500% | 10/15/06 | 5.107% | 106.766 |
| 5.500% | 02/15/08 | 5.157% | 101.996 |
| 6.500% | 02/15/10 | 5.251% | 108.871 |
| 11.250% | 02/15/15 | 5.483% | 155.855 |
| 6.125% | 08/15/29 | 5.592% | 107.551 |

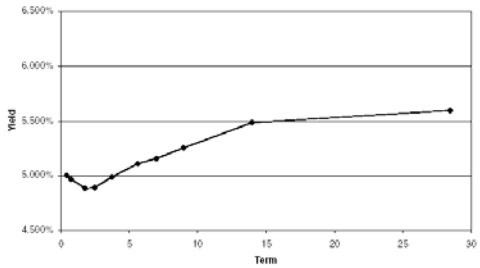


**FIGURE 4.1** Linear Yield Interpolation in the Treasury Market on February 15, 2001

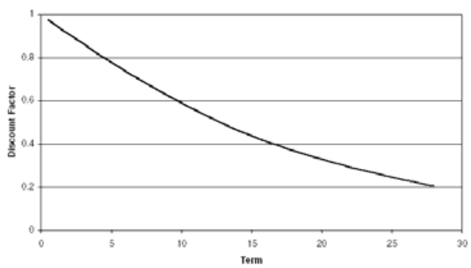


**FIGURE 4.2** The Discount Function from the Linear Yield Interpolation of Figure 4.1

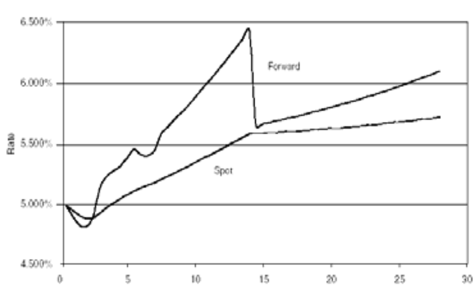


**FIGURE 4.3** Spot and Forward Rates from the Linear Yield Interpolation of Figure 4.1

- Kinks on the spot rate curve, especially at about 14 years.
- Beyond a certain point, like five years, spot rate curves should be concave.
- If a line connecting two points on a curve is below the curve, the curve is said to be concave.
- If the line is above the curve, the curve is convex.

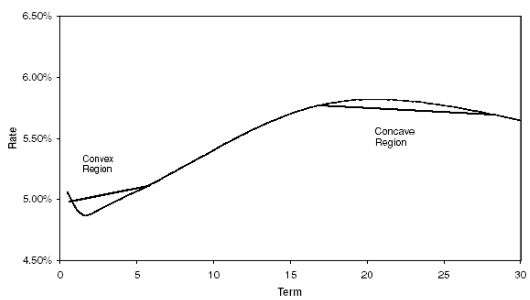


**FIGURE 4.4** Convex and Concave Regions of a Curve

## Forward curve in Fig. 4.3

- Shortcomings of the forward rate curve are obvious.
- Shortcomings:
- discount function < spot rate curve < forward rate curve.

- On Feb 15, 2001, the yields or spot rates on 9.5-year and 10-year C-STRIPS were 5.337% and 5.385%.
- The implied 6-month rate 9.5 years forward is 6.299%.
- Were the yield on the 10-year C-STRIPS to be one basis point lower, at 5.375%, a change of less than .2%, its price would rise by less than .1%.
- The forward rate would fall by 20.1 bp to 6.098%, a change of 3.2%.

## Piecewise Cubics

- To build a smooth curve, assume a functional form for the discount function, for spot rates, or for forward rates.
- For example, use a cubic polynomial:

$$d(t) = 1 + at + bt^2 + ct^3$$

## Piecewise Cubics

- A cubic polynomial for spot rate:

$$\hat{r}(t) = \hat{r}_0 + at + bt^2 + ct^3$$

- Even better, a cubic polynomial for each segment of the curve, then connect all the segments smoothly:
- Piecewise cubic polynomial