Homework Assignment # 2
Math 560
Kaul
Spring 2013
Due Wednesday, May 1

Instructions: To receive full credit, each solution must be neat and legible. Explain your reasoning fully and use complete sentences – an answer without an explanation will receive no credit. Staple the homework sheet to the front of your work.

1. Garling p. 47 # 4.8
2. Garling p. 62 # 7.2
3. Garling p. 67 # 7.4
4. Garling p. 70 # 7.7
5. Extensions $L : K$ and $L' : K'$ are **isomorphic** if there exist field isomorphisms $\lambda : K \rightarrow K'$ and $\mu : L \rightarrow L'$ such that the diagram below commutes.

\[
\begin{array}{c}
K \xrightarrow{\lambda} K' \\
inc \downarrow \quad \downarrow \quad inc \\
L \xrightarrow{\mu} L'
\end{array}
\]

Assume that $L : K$ and $L' : K'$ are isomorphic. Prove the following:

(a) $[L : K] = [L' : K']$
(b) If $L : K$ is separable, then $L' : K'$ is separable.

6. Garling p. 81 # 95

7. Find the normal closure of the following extensions.

(a) $\mathbb{Q}(\sqrt[3]{3}) : \mathbb{Q}$
(b) $\mathbb{Q}(\sqrt{2}, \sqrt{3}) : \mathbb{Q}$
(c) $\mathbb{Q}(\sqrt{2}, \sqrt[3]{2})$

8. Let $\Sigma$ be the splitting field for $x^4 + 1$ over $\mathbb{Q}$. According to Corollary 3.9 there exists an automorphism $\sigma : \Sigma \rightarrow \Sigma$ that fixes $\mathbb{Q}$ and $\sigma(e^{\pi i/4}) = e^{5\pi i/4}$. Give an explicit formula for $\sigma(\alpha)$, where $\alpha$ is an arbitrary element of $\Sigma$. 