Homework Assignment # 6

Math 481
Kaul
Fall 2016
Due Thursday, November 10

Instructions: To receive full credit, each solution must be neat and legible. Explain your reasoning fully and use complete sentences when appropriate – an answer without an explanation will receive no credit. Staple the homework sheet to the front of your work.

1. Fraleigh p. 133 # 6
2. Fraleigh p. 133 # 13
3. Fraleigh p. 133 # 18
4. Fraleigh p. 134 # 21
5. Prove the Universal Mapping Property for $\mathbb{Z} \times \mathbb{Z}$: If $g$ and $h$ are any elements of a group $G$, then there exists a unique homomorphism $\phi : \mathbb{Z} \times \mathbb{Z} \to G$ such that $\phi(1, 0) = g$ and $\phi(0, 1) = h$ if and only if $gh = hg$.
6. Fraleigh p. 134 # 24
7. Fraleigh p. 135 # 50
8. How many homomorphisms $\mathbb{Z}_{25} \to \mathbb{Z}_{10}$ are there? Of these, which ones are epimorphisms? Justify your answers.
9. For a group $G$ let $\text{Aut}(G)$ be the set of automorphisms of $G$.
   (a) Prove that $\text{Aut}(G)$ is a group under composition.
   (b) Compute $\text{Aut}(\mathbb{Z})$ (i. e., prove that $\text{Aut}(\mathbb{Z})$ is isomorphic to a very familiar group).
   (c) Compute $\text{Aut}(\mathbb{Z}_8)$ (ditto).
10. Let $G$ be a group and let $g \in G$.
    (a) Prove that the mapping $\phi_g : G \to G$ given by $\phi(x) = gxg^{-1}$ for all $x \in G$ is an automorphism of $G$. ($\phi_g$ is called an inner automorphism.)
    (b) Prove that the set $\text{Inn}(G)$ of all inner automorphisms of $G$ is a subgroup of $\text{Aut}(G)$. 