Homework Assignment # 1

Math 441/541
Kaul
Spring 2015
Due Monday, April 20

Instructions: To receive full credit, each solution must be neat and legible. Explain your reasoning fully and use complete sentences – an answer without an explanation will receive no credit. Staple the homework sheet to the front of your work.

1. Prove that the set \(\{a_0, \ldots, a_n\} \subseteq \mathbb{R}^N\) is geometrically independent if and only if the set \(\{a_1 - a_0, \ldots, a_n - a_0\}\) is a linearly independent set of vectors.

2. Let \(T : \mathbb{R}^N \to \mathbb{R}^N\) be an affine transformation (a nonsingular linear transformation followed by a translation). If \(\{a_0, \ldots, a_n\} \subseteq \mathbb{R}^N\) is geometrically independent, show that \(\{T(a_0), \ldots, T(a_n)\}\) is geometrically independent.

3. Let \(K\) be a simplicial complex. Show that

\[ A \subseteq |K| \text{ is closed in } |K| \iff A \cap \sigma \text{ is closed in } \sigma \]

defines a topology on \(|K|\).

4. Let \(K\) be the simplicial complex indicated.

   \[ a_0 \quad a_1 \quad a_2 \quad a_3 \quad a_4 \]

   (a) Sketch \(\overline{a_1 a_3}\).
   (b) Sketch \(Lk a_3\).
   (c) Let \(\sigma\) be the 4-simplex in \(\mathbb{R}^4\) spanned by the set
   \[
   \{(0,0,0,0),(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}
   \]
   and let \(\Delta^4\) be the simplicial complex consisting of the faces of \(\sigma\). Find a subcomplex \(L \subseteq \Delta^4\) and an isomorphism \(f : K \to L\).

5. Sketch the geometric realization of the abstract simplicial complex

   \[ S = \{\{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{a, b\}, \{a, c\}, \{b, c\}, \{c, d\}, \{c, e\}, \{d, e\}, \{a, b, c\}, \{c, d, e\}\}. \]

6. Munkres, p. 19 # 1

7. Munkres, p. 20 # 2