Homework Assignment # 2

Math 248
Kaul
Fall 2017
Due Friday, Sept 29

Instructions: To receive full credit, each solution must be neat and legible. Explain your reasoning fully and use complete sentences when appropriate – an answer without an explanation will not receive full credit. Staple the homework sheet to the front of your work.

1. Let $S = \mathcal{P} \{1, 2, 4\}$ and let $P(A) : A \subseteq \{1, 2, 3\}$ be an open sentence over the domain $S$.
   (a) Find all $A \in S$ such that $P(A)$ is true.
   (b) Find all $A \in S$ such that $P(A)$ is false.
   (c) Find all $A \in S$ such that $A \cap \{1, 2, 3\} = \emptyset$.

2. Let $B = \{1, 2, 3, 4, 5, 6\}$ and let
   
   $P(A) : A \cap \{2, 4, 6\} = \emptyset$
   
   $Q(A) : A \neq \emptyset$

   be open sentences over the domain $S = \mathcal{P}(B)$.
   (a) Find all $A \in S$ for which $P(A) \land Q(A)$ is true.
   (b) Find all $A \in S$ for which $P(A) \lor (\sim Q(A))$ is true.
   (c) Find all $A \in S$ for which $P(A) \Rightarrow Q(A)$ is true.

3. Consider the statements

   $P : \sqrt{2}$ is rational
   
   $Q : \frac{2}{3}$ is rational
   
   $R : \sqrt{3}$ is rational

   Write each of the following statements in English and determine their truth values.
   (a) $(P \land Q) \Rightarrow R$
   (b) $((\sim P) \lor Q) \Rightarrow R$

4. Let $P, Q$ be statements. Show that $(P \land (P \Rightarrow Q)) \Rightarrow Q$ is a tautology.

5. Verify DeMorgan’s Laws (Theorem 2.1 (d)).
6. Find the set of all \( x \in S \) for which \( P(x) \Rightarrow Q(x) \) is true.

(a) \( P(x) : x - 3 = 4 \)
    \( Q(x) : x \geq 8 \)
    \( S = \mathbb{R} \)

(b) \( P(x) : x^2 \geq 1 \)
    \( Q(x) : x \geq 1 \)
    \( S = \mathbb{R} \)

(c) \( P(x) : x \in [-1, 2] \)
    \( Q(x) : x^2 \leq 2 \)
    \( S = [-1, 1] \)

7. Determine the truth value of the following statements.

(a) \( \exists x \in \mathbb{Q}, x^2 - 2 = 0 \)
(b) \( \exists x \in \mathbb{Z}, \forall y \in \mathbb{R}, x + y = 1 \)
(c) \( \forall x \in \mathbb{Z}, \exists y \in \mathbb{R}, x + y = 1 \)
(d) \( \exists x \in \mathbb{Z}, \exists y \in \mathbb{Z}, x^2 + y^2 = 9 \)

8. For each statement \( P \) below, (i) express \( P \) in English. (ii) express \( \sim P \) symbolically and (iii) determine the truth value of \( P \).

(a) \( P : \forall x \in (-1, 1), \exists y \in \mathbb{N}, |x + y| < 1 \)
(b) \( P : \exists x \in (-1, 1), \exists y \in \mathbb{N}, |x + y| < 1 \)
(c) \( P : \forall x \in (-1, 1), \exists y \in \mathbb{N}, (|x + y| < 1 \Rightarrow y \geq 1) \)
(d) \( P : \forall x \in (-1, 1), \exists y \in \mathbb{N}, (x^2 < 1 \Rightarrow y < 0) \)