Algebra Tips # 1:

- Develop good algebra habits. One example is instead of writing \(-x + 1\), consider \(1 - x\) or \(-(x - 1)\). Avoiding a \(-\) in front of an expression often reduces the risk of making errors.
- Before attacking a problem, see if changing it with valid operations reduces the risk of errors. For example: multiplying an expression with \(\left(\frac{-1}{-1}\right)\) is like multiplying the expression by one which of course leaves it unchanged. Using this technique, one can change fractions like \(\frac{2x-x^2+4}{2-x^2}\), into \(\frac{x^2-2x-4}{x^2-2}\), which often makes further work easier to do and with less risk of errors.
- Think of a negative sign in front of a fraction as multiplying the fraction by either one of the following: \(\left(\frac{-1}{1}\right)\) = \(\left(\frac{1}{-1}\right)\). This means the negative can multiply the numerator or the denominator, not both.

Algebra Tips # 2:

- Exponents add when multiplying: \(x^{1/2} x^{-3/2} = \frac{1}{x^{1/2} \times x^{3/2}} = x^{-1/2}\)
- Exponents change signs when moving between numerator and denominator.
- Watch for multiple terms inside radicals, or raised to any power. For example \(\sqrt{x^2 + a^2}\) is not equal to \(x + a\); algebra offers no way to simplify this expression.
- Change radicals to fractional exponents, and do algebraic simplifications when possible before attacking a problem since this often reduces the risk of making errors.

Algebra Tips # 3:

- Memorize the special products: \((a \pm b)^2 = a^2 \pm 2ab + b^2\), and \(a^2 - b^2 = (a - b)(a + b)\), and look for them in both directions. This increases your speed and minimizes the risk of errors. NOTE that \(a^2 + b^2\), this is an irreducible quadratic (factors are complex numbers).
- Factoring any term out of an expression is very important. For examples: 1) Factor \(n^3\) from \((n^2 + n^3) = n^3 (1/n + 1);\) 2) Factor \(x\) from \(\sqrt{x^2 + a^2} = |x|\sqrt{1 + (a/x)^2}\)
- Watch for errors from negative exponents in fractions. For example: \(\frac{1}{5 + x^{-2}}\) can be confusing. The form of this fraction can be useful when calculating limits, but recognize equivalent forms: \(\frac{1}{5 + x^{-2}} = \frac{1}{5 + 1/x^2} = \frac{1}{(5x^2 + 1)/x^2} = \frac{x^2}{5x^2 + 1}\). You should feel comfortable manipulating any one of these forms into the other forms.
Algebra Tips # 4:

- A factoring example useful when doing trigonometric substitution is the following: \( \sqrt{4 + x^2} = \sqrt{4(1 + x^2/4)} = 2\sqrt{1 + (x/2)^2} \). Can you readily see the steps?

- Can you prove these are all equivalent fractions: \( \frac{e^{-3x} - e^{3x}}{e^{-3x} + e^{3x}} = \frac{e^{-6x} - 1}{e^{-6x} + 1} = \frac{1 - e^{6x}}{1 + e^{6x}} \). This type of algebraic manipulation of fractions is very important when calculating limits.

- Completing the Square: manipulating a quadratic polynomial into a perfect square plus another term, for example: \( x^2 - x + 4 = [x^2 - x + (1/2)^2] - (1/2)^2 + 4 = (x - 1/2)^2 + 15/4 \)

Differentiation and Algebra Tips # 5:

- \( \frac{d}{dx} (\sqrt[3]{x}) = \frac{d}{dx} (\sqrt[3]{x}) = \sqrt[3]{3} \frac{d}{dx} (x^{1/3}) = \sqrt[3]{3} \left( \frac{1}{2} \right) x^{-2/3} = \frac{\sqrt[3]{3}}{2x^{2/3}} \)

- \( \frac{d}{dx} \left( \frac{x^2 + 3}{4} \right) = \frac{d}{dx} \left( \frac{1}{4} (x^2 + 3) \right) = \frac{1}{4} d \left( x^2 + 3 \right) = \frac{1}{4} (2x) = \frac{x}{2} \)

- \( \frac{d}{dx} \left( \frac{3}{x^4} \right) = \frac{d}{dx} (3x^{-4}) = 3 \frac{d}{dx} (x^{-4}) = 3(-3)x^{-5} = -\frac{9}{x^5} \)

- \( \frac{d}{dx} \left( \sqrt[5]{x^3} \right) = \frac{d}{dx} \left( \sqrt[5]{x^3} \right) = \sqrt[5]{5} \frac{d}{dx} (x^{-3}) = \sqrt[5]{5} \left( -\frac{3}{2} \right) x^{-5/2} = -\frac{3\sqrt[5]{5}}{2x^{5/2}} \)

- \( \left\{ \begin{array}{l} \frac{d}{dx} \left( x^2 + 3 \right) = \frac{d}{dx} \left( \frac{1}{\sqrt{x}} (x^2 + 3) \right) = \frac{d}{dx} \left( x^{-1/2} (x^2 + 3) \right) \\ = \frac{d}{dx} \left( x^{1/2} + 3x^{-1/2} \right) = \left( \frac{3}{2} \right) x^{1/2} + 3 \left( -\frac{1}{2} \right) x^{-1/2} = \frac{3}{2} (x^{1/2} - x^{-1/2}) \end{array} \right\} \)

- \( \left\{ \begin{array}{l} \frac{d}{dx} \left( \frac{x^2 + 3}{\sqrt{x}} \right) = \frac{d}{dx} \left( \frac{x^2 + 3}{x^{1/2}} \right) = \frac{x^{1/2} (2x) - (x^2 + 3) \left( \frac{1}{2} \right) x^{-1/2}}{x} \\ = \frac{2x^{3/2} - (1/2)x^{3/2} - (3/2)x^{1/2}}{x} = \frac{(3/2)x^{3/2} - (3/2)x^{1/2}}{x} = \left( \frac{3}{2} \right) x^{1/2} - \left( \frac{3}{2} \right) x^{-1/2} \end{array} \right\} \)