DISCRETE SYSTEMS: The Dripping Faucet

Water dripping from a faucet, which is a common occurrence in life, displays many features of nonlinear systems including chaotic transitions, bifurcations, and multiple basins of attraction. Because of this complex behavior and its easy accessibility, investigating the dynamics of water dripping at different flow rates is a classic experiment. As the flow rate is varied the system can change from periodic to chaotic behavior. In this experiment we will measure the time interval between successive drops at different flow rates and learn techniques to quantify the observed behavior. Although it might not be of much practical use to quantify the behavior of water drops, these techniques can be applied to more complex problems such as cardiac rhythms, exchange rates, etc.

Our system consists of a water tank with a supply line and two outlet lines (one leading to a supply tank and the other to a thin pipette), a pump, a drain tank, a photogate connected to the computer, and a bucket. The supply line is connected to a small pump which pumps the water back up to the water tank from the supply tank and the drain line drains the water tank when the water reaches a certain level. This maintains the water in the tank at a constant level providing a constant pressure. The hose leading to the pipette has a valve to adjust the flow rate. The flow rate can also be adjusted by moving the tank vertically relative to the pipette. The photogate is placed so that drops fall in front of the infrared beam which goes from arm to arm of the photogate. This starts a timer that stops when the next drop comes through and thus gives you the elapsed time between successive drops.

1. Setting up:

a) Adjust the system so that water drops fall in front of the beam of the photogate and the pipette is vertical. Check that none of your tubes are pinched. Nonlinear systems are quite susceptible to small perturbations. **Throughout the data run be very careful not to perturb the apparatus!**

b) Open the Logger Pro file DrippyFaucet from the desktop. Click on the Collect button and you should be able to see the interval between drops on the screen. Note that the photogate will be continuously monitored until you click Stop.

c) You can adjust the valve on the tube leading to the pipette and/or move the tank so that the time interval between successive drops is roughly 100-300 milliseconds. You may need to adjust the vertical scale of the plot-this can be done while data is being collected.
2. Experiment:

Adjust the valve and/or move the water tank on the rod to change the flow rate and observe how the time intervals change. Notice that when the system is periodic, with one period, the dots form a steady horizontal line since the time interval between successive drops is always the same. When the drop rate starts fluctuating slightly there will be some variation in the time intervals. When the system is in a "period doubled" state you see two different time intervals alternating, every other pair of drops has the same time interval. You should check the raw tabulated data to make sure that the time intervals alternate from step to step. Note that the period is not actually doubled. It just means that there are two different periods describing the drop intervals.

♦ Find flow rates that display the following behavior, and collect, save, and print data as explained below.

- periodic
- periodic with small fluctuations (~5--10%)
- period doubled
- chaotic

3. Collecting, saving and printing data:

a) When you have a flow rate that gives data exhibiting one of the above behaviors, stop the data collection. From the File menu choose Export, and save the data to your folder on the Desktop with an informative name.

b) Print out the graph of the data. Scale your axes so the data looks reasonable and make sure you give the graphs a descriptive title.

4. Analyzing data:

Return map: One useful way of analyzing the data is by generating a "return map", which is a plot of successive time intervals, \( \Delta t_{n+1} \) vs. \( \Delta t_n \). This is most easily accomplished in Excel.

a) Go to the "raw data" window in Logger Pro and click on the Delta T column, it should become highlighted. Now copy the data by using control-C. (Note that you can also highlight data by clicking on the plot and dragging over the area of interest).

b) Open Excel, click in the second row of column A, use control-V to paste the data here.

c) Now move to the first row of column B, use control-V to copy it here as well.

d) Select the data and generate a plot using the Chart button. Select x-y scatter. Adjust the axis scales so that they are the same (e.g. 0 to 200 ms on each axis).

Histogram: Another useful view comes from generating a histogram of all the time intervals between successive drops. You can do this in Excel using the Histogram feature in Tools/Data Analysis.
For each type of behavior you should have the following graphs:

Note: Do not connect the data points with lines when you plot the following graphs!

a) $\Delta t$ versus $t$, this simple graph shows the time intervals as they occurred. You should have generated this using the Logger Pro software.
b) Return map (this is the analog of a Poincare section), $\Delta t_{n+1}$ versus $\Delta t_n$.
c) Histogram of time intervals $\Delta t_n$.

5. Questions:

1) What is the fundamental difference between this system and the other systems that we have previously analyzed?

2) For the data set corresponding to each type of behavior explain how each of the above graphs quantifies the behavior.

3) What aspects of the histogram provide signatures for distinguishing between different types of behavior? Sketch the histogram you would expect to see for a period quadrupled system.