SUPERCRITICAL PITCHFORK BIFURCATION: Bead on a Hoop

In section 3.5 of Strogatz you studied the over-damped bead on a hoop. In this lab we will do an experiment to quantitatively verify some of the predictions of the model. You should review the derivation of the equation of motion (equations (1) and (2) in sect. 3.5) and the location of fixed points as a function of the parameter $\gamma = \frac{r\omega^2}{g}$.

As part of your lab report, do and turn in a linear stability analysis of the fixed points (this is exercise 3.5.2).

The experimental apparatus comprises a rotating hoop with a rail inside along which a ball bearing can roll (see Figure below). One difference from the treatment in Strogatz that you'll notice is that our apparatus is not overdamped so that oscillations can occur. However, we will not explore such oscillations. Therefore, we will be able to use the analysis in Strogatz since we'll only be looking at the system in equilibrium, i.e. $\dot{\phi} = \ddot{\phi} = 0$. The rotation speed of the motor driving the hoop can be varied using the power supply. This corresponds to varying the parameter $\omega$ and will lead to different equilibrium angles for the ball.

Experiment

The radius of the hoop (i.e. from the hoop center to the center-of-mass of the ball bearing) is 6.7 +/- 0.2 cm.

1. ✺ Calculate the critical frequency at which the bifurcation occurs.
2. Initially, without taking detailed data, change the driving speed and get a sense of the behavior of the system. You will probably notice that most of the “action” (the interesting behavior) takes place somewhere between about 1.5 and 3 Hz. Does this make sense? Compare your qualitative observations with the structure shown in Figure 3.5.6 in Strogatz. Start with the hoop at rest and slowly increase the rotation rate. When the ball leaves the bottom which side does it go to? always?
3. To quantitatively analyze the system we'll measure the equilibrium angle, $\phi^*$ (the fixed point), of the ball as a function of rotation frequency. To do this we'll make a movie of the hoop in rotation and then use frame-by-frame capability of the software to freeze the motion and measure the equilibrium angle.
4. It turns out that it's easier to map out the bifurcation diagram by starting at high frequency and coming down. Once you are comfortable with the apparatus, raise the rotation rate to about 3 Hz and measure the period/frequency using "BeadHoop" in LoggerPro. (Or you can simply touch your finger lightly on the axle of the hoop and count rotations for 30 seconds or so.) Be sure you have a record of the frequency of rotation, write it on a piece of paper and place the paper in the field of view of the camera. (NOTE: It sometimes takes a while for the hoop to settle down at a new frequency, using BeadHoop be sure the frequency is essentially constant before recording the motion.)
5. Start the video camera and record the hoop/ball motion for about 10 seconds. Save this movie. (Detailed instructions making movies are in the lab.) Now reduce the frequency slightly, measure it, indicate the frequency by putting a new piece of paper in the field of view, and record another 10 seconds of movie. Save this movie and then repeat this procedure until the ball is at the bottom of the hoop and you have at least 10 data points.
6. Examine your movie with the frame controls (forward/backward/pause) to find a frame from which you can measure the angle of the ball. The edge of the hoop has been marked with ticks spaced $5^0$ apart. The data is best if you find a frame with most of the hoop in focus and parallel to the screen. As you do the experiment, try and get some feeling for the uncertainty associated with the measurements. For example, how accurately is the frequency measured? How well can you judge the angles?

Questions:

♦ **Along with the diamond items above, responses to each of the following should be in your report.**

1. Plot your experimental $\phi^*$ vs. $\gamma$ and show where the bifurcation seems to occur. Superimpose a plot of the theoretical $\phi^*$ vs. $\gamma$.
2. Since it's probable that the bifurcation will not appear exactly where expected, explain where you think any disagreement comes from. Consider the role of an imperfection parameter in this system.
3. Does the ball always go up the same side of the hoop or does it go up either side with equal probability? What “breaks the symmetry”? If the ball always goes up the same side, can we still use this apparatus to verify the model?

References
If you would like to pursue this experiment in more detail, ask your instructor for a copy of “A mechanical analog of first- and second-order phase transitions” by G. Fletcher, *American Journal of Physics*, 65 (1), 74-81 (Jan 1997).